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Rohini

GATE SYLLABUS

CONTROL SYSTEMS (ECE & EEE)

Basic control system components: block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feed back) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control systems analysis: root loci,

Routh – Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional - Integral – Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

CONTROL SYSTEMS

CONTENTS

For

GATE, DRDO & IES

Managing Director
Y.V. Gopala Krishna Murthy

System: A system is an arrangement or combination of different physical components such that it gives the proper output to given input. A kite is an example of a physical system, because it is made up of paper and sticks. A classroom is an example of a physical system.

Control: The meaning of control is to regulate, direct or command a system so that a desired objective is obtained.

Plant: It is defined as the portion of a system which is to be controlled or regulated. It is also called a process

Controller: It is the element of the system itself, or may be external to the system. It controls the plant or the process.

Input: The applied signal or excitation signal that is applied to a control system to get a specified output is called input.

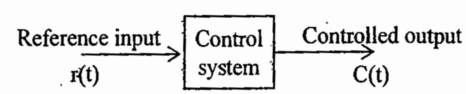
Output: The actual response that is obtained from a control system due to the application of the input is termed as output.

Disturbances: The signal that has some adverse effect on the value of the output of a system is called disturbance. If a disturbance is produced within the system, it is termed as an internal disturbance: otherwise, it is known as an external disturbance.

Control Systems: It is an arrangement of different physical components such that it give the desire output for the given input by means of regulate or control either direct or indirect method.

A control system must have (1) input, (2) output, (3) ways to achieve input and output objectives and (4) control action.

Fig. The following shows the cause-and-effect relationship between the input and the output.



Any system can be characterized mathematically by (1) Transfer function (2) State model

$$\text{Transfer Function} = \left.\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}\right|_{\text{initial conditions} = 0}$$

$$= \left.\frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)}\right|_{\text{initial conditions} = 0}$$

Transfer function is also called impulse response of the system.

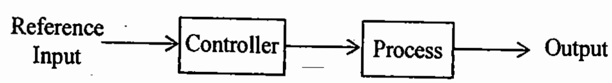
$$C(s) = T.F. \times R(s)$$

Classification of Control System :

- Open-loop C.S
- Closed-loop (or) feedback C.S
  1) Positive feedback C.S
  2) Negative feedback C.S

Open-loop Control System :

The Open-loop control system can be described by a block diagram as shown in the figure.



The reference input controls the output through a control action process. In the block diagram shown, it is observed the output has no effect on the control action. Such a system is termed as open-loop control system.

In an open-loop control system, the output is neither measured nor fed-back for comparison with the input. Faithfulness of an open-loop control system depends on the accuracy of input calibration.

Examples for open – loop control systems are traffic lights, fans, any system which is not having the sensor.

Advantages and disadvantages of open – loop system:

Advantages

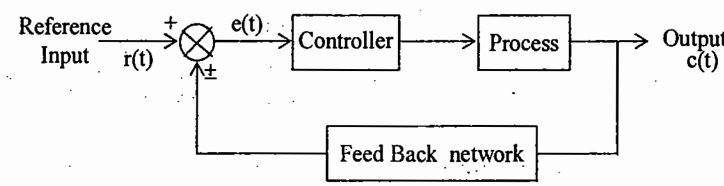
- These systems are simple in construction and design.
- These systems are economic.
- These systems are easy from maintenance point of view.
- Usually these systems are not much troubled with problems of stability.
- These systems are convenient to use when output is difficult to measure.

Disadvantages

- These systems are not accurate and reliable because their accuracy is dependent on the accuracy of calibration.
- In these systems, inaccurate results are obtained with parameter variations, i.e., internal disturbances.
- Recalibration of the controller is required from time to time for maintaining quality and accuracy

Closed-loop Control System :

In a closed-loop control system, the output has an effect on control action through a feedback as shown and hence closed-loop control systems are also termed as feedback control systems. The control action is actuated by an error signal 'e(t)' which is the difference between the input signal 'r(t)' and the output signal 'c(t)'. This process of comparison between the output and input maintains the output at a desired level through control action process.



The control systems without involving human intervention for normal operation are called automatic control systems. A closed-loop (feedback) control system using a power amplifying device prior to controller and the output of such a system being mechanical i.e., position, velocity, acceleration is called servomechanism.

Advantages and disadvantages of closed – loop system

Advantages

- In these systems accuracy is very high due to correction of any arising error.
- Since these systems sense environmental changes as well as internal disturbances, the errors are modified.
- There is reduced effect of non – linearity in these systems.
- These systems have high bandwidth, i.e., high operating frequency zone.
- There are facilities of automation in these systems.

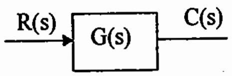
Disadvantages

- These systems are complicated in design and, hence, costlier.
- These systems may be unstable.

Comparison of Open-loop and Closed-loop Control systems :

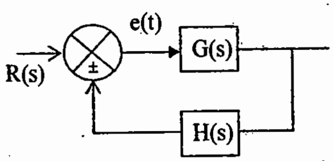
Open – loop C.S.	Closed – loop C.S.
1. The accuracy of an open-loop system depends on the calibration of the input. Any departure from pre – determined calibration affects the output.	1. As the error between the reference input and the output is continuously measured through feedback, the closed – loop system works more accurately.
2. The open – loop system is simple to construct and cheap	2. The closed – loop system is complicated to construct and costly
3. The open – loop systems are generally stable.	3. The closed – loop systems can become unstable under certain conditions
4. The operation of open – loop system is affected due to presence of non linearity's in its elements.	4. In terms of the performance, the closed – loop systems adjusts to the effects of non – linearity's present in its elements.

Open-loop C.S. :

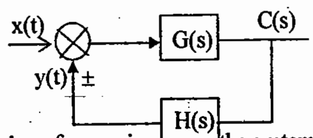


$$\frac{C(s)}{R(s)} = G(s) \quad \text{or} \quad C(s) = G(s)\,R(s)$$

Closed-loop C.S. :



If error signal e(t) is zero, output is controlled.
If error signal is not zero, output is not controlled.



For Positive feedback, error signal = x(t) + y(t)

For Negative feedback, error signal = x(t) - y(t)

The purpose of feedback is to reduce the error between the reference input and the system output.

+Ve feedback: Unity F/B (H(s) = 1) ; Non unity F/B(H(s) ≠ 1)

-Ve feedback: Unity F/B ; Non unity F/B

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)}; \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}; \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}; \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

Where G(s) = T.F without feedback (or) T.F of the forward path

H(s) = T.F of the feedback path

The feedback has effects on such system performance characteristics as stability, bandwidth, overall gain, impedance and sensitivity.

1.1 Effect of feedback on Stability :

Stability is a notion that describes whether the system will be able to follow the input command. A system is said to be unstable, if its output is out of control or increases without bound. It can be demonstrated that one of the advantages of incorporating feedback is that it can stabilize an unstable system.

Effect of feedback on Overall gain :

Feedback affects the gain G of a non-feedback system by a factor of 1 ± GH. The general effect of feedback is that it may increase or decrease the gain. In a practical control system, G and H are functions of frequency, so the magnitude of 1 + GH may be greater than 1 in one frequency range but less than 1 in another. Therefore, feedback could increase the gain of the system in one frequency range but decrease it in another.

1.2 Effect of feedback on Sensitivity:

Consider G as a parameter that may vary. The sensitivity of the gain of the overall system M to the variation in G is defined as

$$S_G^M = \frac{\partial M / M}{\partial G / G}$$

where ∂M denotes the incremental change in M due to the incremental change in G; $\partial M / M$ and $\partial G / G$ denote the percentage change in M and G, respectively.

$$S_G^M = \frac{\partial M}{\partial G}\,\frac{G}{M} = \frac{1}{1 + GH}$$

This relation shows that the sensitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable. In an open-loop system, the gain of the system will respond in a one-to-one fashion to the variation in G. In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located.

The effects of feedback are as follows.

(i)Gain is reduced by a factor
(ii) There is reduction of parameter variation by a factor 1 + G(s)H(s).
(iii) There is improvement in sensitivity.
(iv) There may be reduction of stability.

The disadvantages of reduction of gain and reduction of stability can be overcome by gain amplification and good design, respectively.

For a complicated system, it is easy to find the transfer function of each and every element, and output of a certain block may act as an input to other block or blocks. Therefore, the knowledge of transfer function of each block is not sufficient in this case. The interrelation between the elements is required to find the overall transfer function of the system. There are two method : (1) Block diagram and (2) signal flow graph.

CHAPTER - 2 BLOCK DIAGRAM S AND SIGNAL FLOW GRAPHS

There are two methods: (1) by using Block diagram or (2) Signal flow graph, to find the overall transfer function of a big complicated control system.

2.1 BLOCK DIAGRAM ALGEBRA

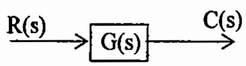
Block diagram reduction techniques:

Some of the important rules for block diagram reduction techniques are given below :

1. The block diagram shown below relates the output and input as per the transfer function relation given below :

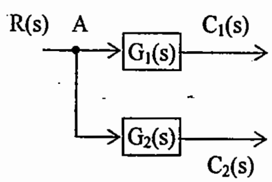
$$G(s) = \frac{C(s)}{R(s)} \qquad \text{(or)} \qquad C(s) = R(s) \cdot G(s)$$

where G(s) is known as the transfer function of the system.



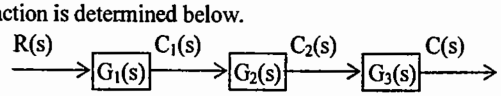
2. **Take off point :**

Application of one input source to two or more systems is represented by a take off point as shown at point A in the below figure.



3. **Blocks in cascade :**

When several blocks are connected in cascade, the overall equivalent transfer function is determined below.



$$\frac{C_1(s)}{R(s)} = G1(s)$$

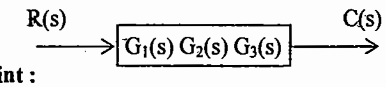
$$\frac{C_2(s)}{C_1(s)} = G2(s)$$

$$\frac{C(s)}{C_2(s)} = G3(s)$$

Multiplying above three equations, the equivalent transfer function is

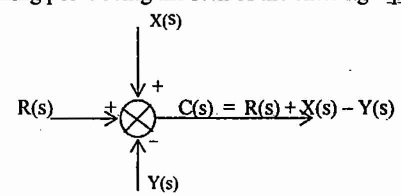
$$\frac{C(s)}{R(s)} = G_1(s)\, G2(s)\, G3(s)$$

The equivalent diagram is given by

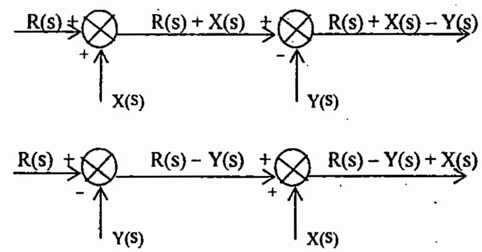


4. **Summing point :**

Summing point represents summation of two or more signal entering in a system. The output of a summing point being the sum of the entering inputs.

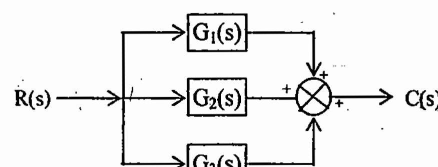


5. **Interchanging summing points:** Consecutive summing points can be interchanged, as this interchange does not alter the output signal.



6. **Blocks in parallel:**

When one or more blocks are connected in parallel, the overall equivalent transfer function is determined below.



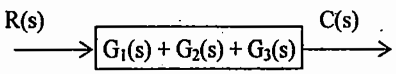
$$C(s) = R(s)\, G_1(s) + R(s)\, G_2(s) + R(s)\, G_3(s)$$

or $$C(s) = R(s)\left\{G_1(s) + G_2(s) + G_3(s)\right\}$$

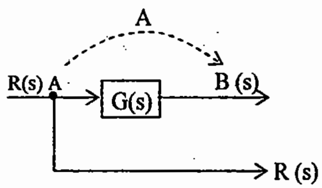
Therefore, the overall equivalent transfer function is,

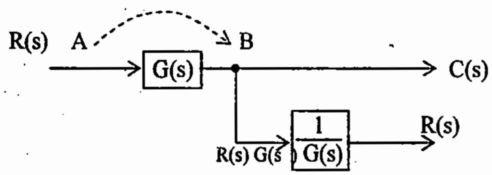
$$\frac{C(s)}{R(s)} = [\, G_1(s) + G_2(s) + G_3(s) \,]$$

The equivalence of above diagram is

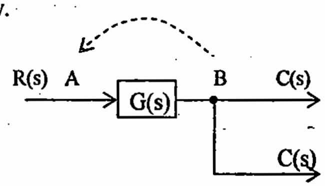


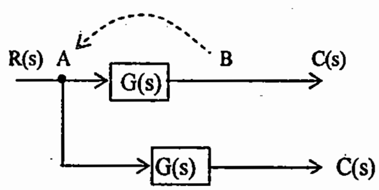
7. Shifting of a take off point from a position before a block to a position after the block is shown below.



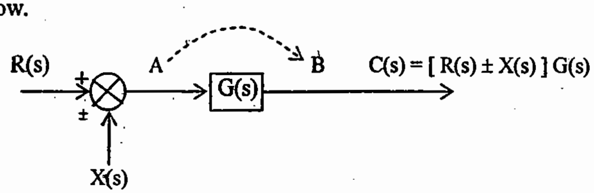


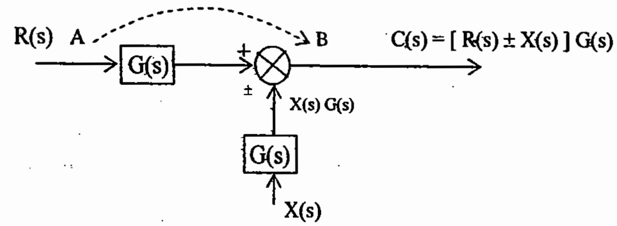
8. Shifting of a take off point from a position after a block to a position before the block is shown below.



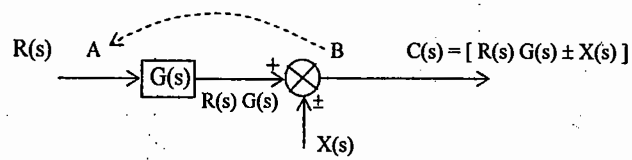


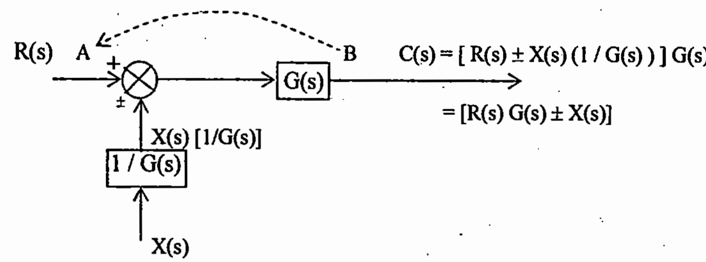
9. Shifting of a summing point from a position before a block to a position after the block is shown below.



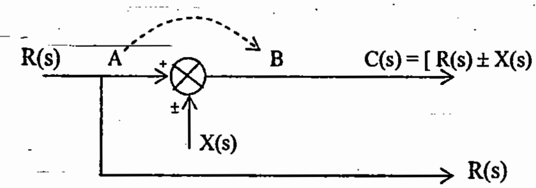


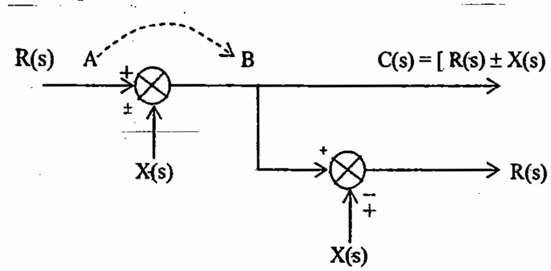
10. Shifting of a summing point from B position after a block to a position A before the block is shown below.



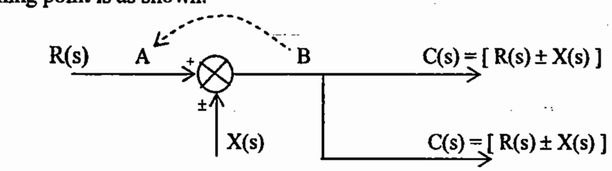


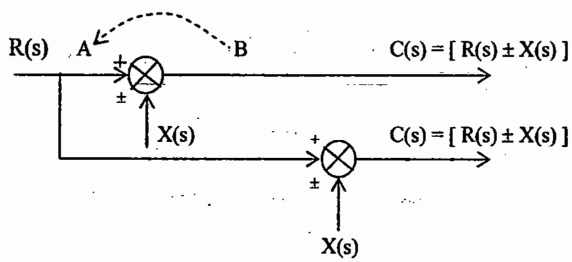
11. Shifting of a take off point from A position before a summing point to a position B after the summing point is shown below.





12. Shifting a take off point from a position after a summing point to a position before the summing point is as shown:





2.2 SIGNAL FLOW GRAPHS

A signal flow graph may be defined as a graphical means of portraying the input-output relationships between the variables of a set of linear algebraic equations.

Consider that a linear system is described by the set of N algebraic equations

$$y_j = \sum_{k=1}^{N} a_{kj}\, y_k \qquad j = 1, 2, \ldots\ldots, N$$

Basic properties of signal flow graphs :

1. A signal flow graphs applies only to linear systems.
2. The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as functions of causes.
3. Nodes are used o represent variable. Normally, the nodes are arranged from left to right, following a success of causes and effects through the system.
4. Signals travel along branches only in the direction described by the arrows of the branches.
5. The branch directing from node y_k to y_j represents the dependence of the variable y_k upon y_j, but not the reverse.
6. A signal Y_k traveling along a branch between nodes y_k and y_j is multiplied by the gain of the branch, a_{kj}, so that a signal $a_{kj}\, y_k$ is delivered at node y_j.

Definitions for Signal Flow Graphs:

Input Node (Source): An input node is a node that has only outgoing branches.

Output Node (Sink): An output node is a node which has only incoming branches.

Path: A path is any collection of a continuous succession of branches traversed in the same direction.

Forward Path: A forward path is a path that starts at an input node and ends at an output node and along which no node is traversed more than once.

Loop: A loop is a path that originates and terminates on the same node and along which no other node is encountered more than once.

Path gain: The product of the branch gains encountered in traversing a path is called the path gain.

Forward path gain: Forward path gain is defined as the path gain of a forward path.

Loop gain: Loop gain is defined as the path gain of a loop.

Masons Gain formula:

The general gain formula is

$$M = \frac{Y_{out}}{Y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

where

M = gain between y_{in} and y_{out}

y_{out} = output node variable

y_{in} = input node variable

N = total number of forward paths

M_k = gain of the k^{th} forward path

$$\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} - \sum_m P_{m3} + \ldots\ldots$$

= 1 – (sum of all individual loop gains) + (sum of gain products of all possible combinations of two non-touching loops) – (sum of the gain products of all possible combinations of three non-touching loops) +……

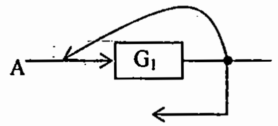
P_{mr} = gain product of the m^{th} possible combination of 'r' non-touching loops

Δ_k = the Δ for the part of the signal flow graph which is non-touching with the k^{th} forward path

Objective Questions

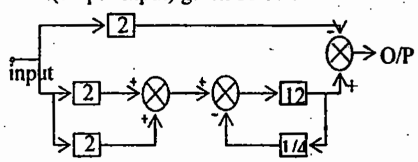
01. The block diagram contains
   (a) system output variable
   (b) system input variable
   (c) the functional relations of the variables
   (d) all the above

02. In a block diagram, when a take off point is moved ahead of a block, G_1.



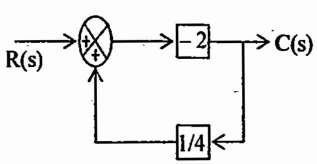
   (a) The block G_1 will be added in parallel.
   (b) The block G_1 will be added in the forward path.
   (c) The block G_1 will be added in series.
   (d) The block G_1 will be added in the feedback path.

03. What is the gain of the system (output/input) given below?



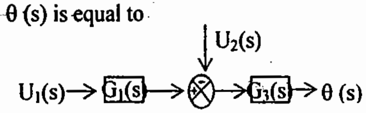
   (a) 36 (b) 10
   (c) 90 (d) –10

04. The closed-loop gain of the system sketched below is



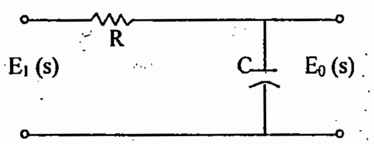
   (a) –4/3 (b) –4
   (c) 4 (d) 4/3

05. In the block diagram shown, the output θ (s) is equal to



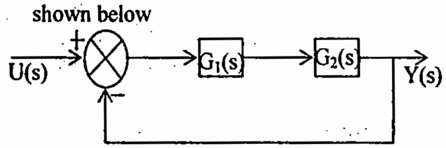
   (a) $U_1(s) + U_2(s)$
   (b) $U_1(s)\,G(s)$
   (c) $G_1(s)\,G_3(s)\,U_1(s) - G_3(s)\,U_2(s)$

06. The transfer function $E_0(s)/E_1(s)$ of the RC-network shown is given by



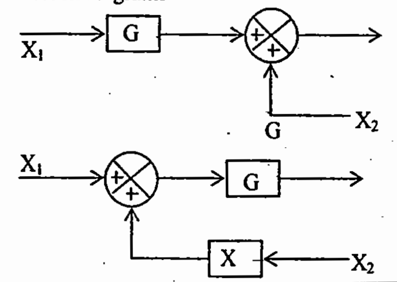
   (a) $\dfrac{1}{RCS+1}$ (b) $\dfrac{1}{RCS}$
   (c) $\dfrac{RCS}{RCS+1}$ (d) None

07. The block diagram of a certain system is shown below



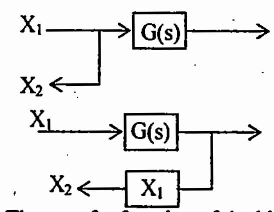
   The transfer function Y(s)/U(s) is given by
   (a) $\dfrac{G_1(s)\,G_2(s)}{1 - G_1(s)\,G_2(s)}$
   (b) $G_1(s)\,G_2(s)$
   (c) $\dfrac{1 + G_1(s)\,G_2(s)}{G_1(s)\,G_2(s)}$
   (d) $\dfrac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)}$

08. The figure below gives two equivalent block diagrams



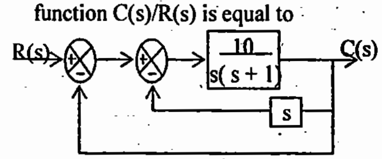
   The value of transfer function of block marked 'X' is given by
   (a) G (s) (b) 1 / G (s)
   (c) 1 (d) 1 + G(s)

09. The figure shows two equivalent block diagrams



   The transfer function of the block marked 'X' is given by
   (a) G(s) (b) 1 / G(s)
   (c) 1 (d) 1 + G(s)

10. For the system shown, the transfer function C(s)/R(s) is equal to



   (a) $\dfrac{10}{s^2+s+10}$ (b) $\dfrac{10}{s^2+11s+10}$
   (c) $\dfrac{10}{s^2+9s+10}$ (d) $\dfrac{10}{s^2+2s+10}$

Key for Objective Questions :

1. d 2. d 3. b 4. a 5. c
6. a 7. d 8. b 9. b 10. b

OBJECTIVE QUESTIONS

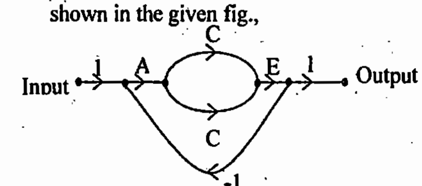
11. In a signal flow graph, the nodes represent
   (a) the system variables
   (b) the system gain
   (c) the system parameters
   (d) all the above

12. The branch of a signal flow graph represents
   (a) the system variable
   (b) the functional relations of the variables
   (c) the system parameters
   (d) none of the above

13. By applying Mason's gain formula, it is possible to get
   (a) the ratio of the output variable to input variable only
   (b) the system functional relations between any two variables
   (c) the overall gain of the system
   (d) the ratio of any variable to input variable only

14. Two or more loops in a signal flow graph are said to be non-touching
   (a) if they do not have any common branch
   (b) if they do not have any common loop
   (c) if they have common node
   (d) if they do not have any common node

15. The transfer function of the system shown in the given fig.,



   (a) $\dfrac{ACE}{1+ACE}$ (b) $\dfrac{ACE}{1-ACE}$
   (c) $\dfrac{2ACE}{1+ACE}$ (d) $\dfrac{2ACE}{1+2ACE}$

16. Signal flow graph is a

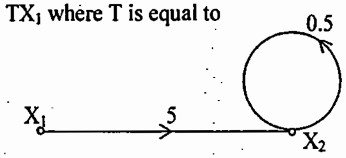
(a) Topological representation of a non-linear differential equation.

(b) Schematic graph.

(c) Special type of graph for analysis of modern control system.

(d) Plot between frequency and magnitude in dB.

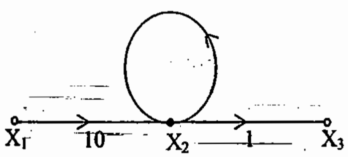
17. The signal flow graph shown, $X_2 = TX_1$ where T is equal to



(a) 2.5 (b) 5

(c) 5.5 (d) 10

18. For the signal flow graph shown in figure, $X_3 / X_1 =$



(a) 10 (b) 15

(c) 20 (d) 25

Key for Objective Questions :

11. a 12. b 13. c 14. d 15. d

16. c 17. d 18. c

OBJECTIVE QUESTIONS:

01. Which of the following is/are the example of the open loop control system?

(A) Metadyne
(B) Field control d.c. motor
(C) An automatic toaster
(D) Both 'B' and 'C'

02. The O/P of a feedback control system must be a function of

(A) reference I/P and output

(B) output

(C) reference I/P

(D) reference I/P and error signal

03. AC control system has the advantage (s) of

(A) availability of rugged high power amplifiers
(B) smaller frame size of a.c components
(C) both A and B
(D) None

04. The transfer function of a system is the Laplace transform if its

(A) ramp response

(B) impulse response

(C) square wave response

(D) step response

05. Any physical system which does not automatically correct for variation on its O/P is called a/an

(A) unstable system

(B) open loop system

(C) closed loops system

(D) None

06. In a control system the comparator compares the O/P response and reference I/P and actuates the

(A) primary sensing element
(B) transducer
(C) signal conditioner
(D) control elements

07. Transfer function of a control system depends on

(A) nature of output
(B) nature of input
(C) initial conditions of input and output
(D) system parameters only

08. The open loop control system is one in which?

(A) O/P is independent of control I/P
(B) only system parameters have effect on the control output
(C) O/P is dependent on control I/P
(D) all of these

09. A unit step function on integration results in a

(A) unit parabolic function
(B) unit doublet
(C) unit step function
(D) unit ramp function

10. The transfer function of a system is defined as the

(A) step response
(B) response due to an exponentially varying input
(C) Laplace transform of the impulse response
(D) All of the above

11. The order of the system is determined by number of

(A) multiplying terms in the denominator
(B) poles at the origin
(C) stable roots of the system
(D) none

12. In a control system an error detector
(A) produces an error signal as actual difference of value and desired value of output
(B) detects the system errors
(C) detects the error and signal out an alarm
(D) none

13. Potentiometers are used in Control system

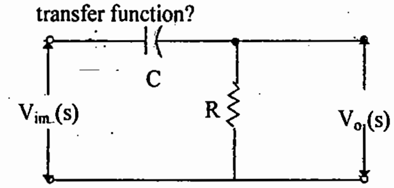
(A) to improve stability
(B) to improve frequency response
(C) as error sensing transducer
(D) to improve time response

14. A control system with excessive noise, I likely to suffer from

(A) loss of gain
(B) vibrations
(C) oscillations
(D) saturation is amplifying stages

15. The type number of a transfer function denotes
(A) the number of poles at infinity
(B) the number of finite poles
(C) the number of zeros at origin
(D) the number of poles at origin

16. For the network given below, what is the transfer function?



(A) $\frac{sRC}{1+sRC}$ (B) s RC + 1

(C) $\frac{RC}{1+sRC}$ (D) $\frac{1}{1+sRC}$

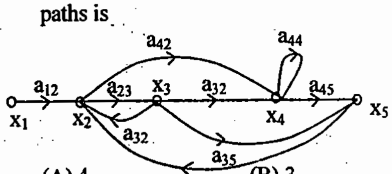
17. The system response can be tested better with _____ signal
(A) exponential decaying
(B) unit impulse input
(C) sinusoidal input
(D) ram input

18. In a control system, the use of negative feedback

(A) increases the influence of variations of component parameters on the system performance
(B) reduces the effects of disturbance and noise signals in the forward path
(C) increases the reliability
(D) eliminates the chances of instability

19. A signal flow graph is a
(A) log log graph
(B) special type of graph to analyse modern control systems
(C) polar graph
(D) topological representation of a set of differential equation

20. The signal flow graph for a system is shown below. The number of forward paths is



(A) 4 (B) 3
(C) 1 (D) 2

KEYS:

01. A	02. D	03. B	04. B	05. B
06. D	07. D	08. A	09. D	10. C
11. B	12. A	13. C	14. D	15. B
16. A	17. B	18. B	19. B	20. B

Previous PSU's Question

01. The Laplace transform of a transportation lag of 5 seconds is
(A) exp (- 5 s) [illegible] (5 s)
(C) $\frac{1}{s+5}$ (C) $\left(-\frac{1}{5}\right)$

02. The impulse response of an initially relaxed linear system is e^{-2t} u (t). To produce a response of te^{-2t} u (t). The input must be equal to
(A) 2 e^{-t} u (t) (B) ½ e^{-2t} u (t)
(C) e^{-2t} u (t) (D) e^{-t} u (t)

03. The Laplace transformation of f(t) is F(s).
Given the $F(s) = \frac{\omega}{s^2+\omega^2}$ final value of f (t) is
(A) inifinity (B) zero
(C) one (D) indeterminate

04. As compared to closed loop system, an open loop is
(A) more stable as well as more accurate
(B) less stable as well as less accurate
(C) more stable but less accurate
(D) less stable but more accurate

05. The impulse response of a system is given by ½ e^{-2t} Which one of the following is its unit step responses.
(A) $1 - e^{-1/2t}$ (B) $1 - e^{-t}$
(C) $2 - e^{-2t}$ (D) $1 - e^{-2t}$

06. Signal flow graph is used to find
(A) stability of the system
(B) controllability of the system
(C) Transfer function of the system
(D) poles of the system

07. The transfer function of a technometer is of the form
(A) Ks (B) $\frac{K}{s}$
(C) $\frac{K}{s+1}$ (D) $\frac{K}{s(s+1)}$

PSU's

(1) A (2) C (3) D (4) C (5) A
(6) C (7) A

The time response has utmost importance for the design and analysis of control systems because these are inherently time domain systems where time is the independent variable. During the analysis of response, the variation of output with respect to time can be studied and it is known as time response. To obtain satisfactory performance of the system, the output behavior of the system with respect to time must be within the specified limits. From time response analysis and corresponding results, the stability of system, accuracy of system and complete evaluation can be studied very easily.

Due to the application of an excitation to a system, the response of the system is known as time response and it is a function of time. There are two parts of response of any system: (i) transient response and (ii) steady- state response.

Transient Response:

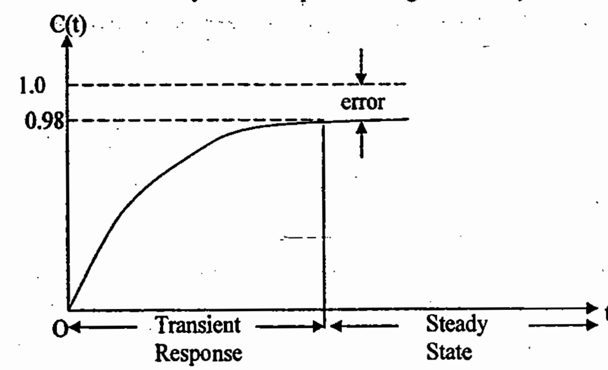
The part of the time response which goes to zero after large interval of time is known as transient response. In this case $\underset{t=\infty}{Lt}\ C(t)_t = 0$. From transient response, we get the following information:
(a) The time interval after which the system responds taking the instant of application of excitation as reference.
(b) The total time that it takes to achieve the output for the first time.
(c) whether or not the output shoots beyond the desired value and how much.
(d) whether or not the output oscillates about its final value.
(e) The time that it takes to settle to the final value.

Steady State Response

The part of response that remains even after the transients have died out is said to be steady-state response. From steady – state response, we get the following information:
(a) The time that output takes to reach the steady – state
(b) Whether or not any error exists between the desired and the actual value.
(c) Whether this error is constant, zero, or infinite.
The total response of a system is the sum of transient response and steady – state response:

$$C(t) = C_{tr}(t) + C_{ss}$$

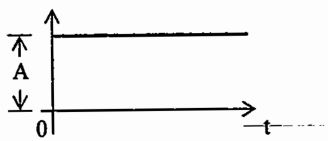
Figure shows the transient and steady – state responses along with steady – state error.



3.1 Transiant analysis

Standard test signals:

(1) Step function : Step function is described as sudden application of input signal as illustrated in figure.



$r(t) = A\,u(t)$

$u(t) = 1\ ;\ t \geq 0$

$= 0\ ;\ t < 0$

In the Laplace transform form, $R(s) = A/s$

Step function /displacement function.

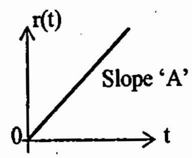
(2) Ramp function : The Ramp is a signal which starts at a value of zero and increases linearly with time. Mathematically,

$r(t) = At\ ;\ \text{for } t \geq 0$

$= 0\ \ ;\ \text{for } t < 0$

In the Laplace transform from, $R(s) = A/s^2$

Ramp function is also called velocity function.

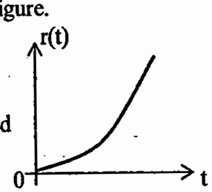


(3) Parabolic function : Parabolic function is described as more gradual application of input in comparison with ramp function as illustrated in figure.

$r(t) = At^2/2\ ;\ \text{for } t \geq 0$

$r(t) = 0\ \ ;\ \text{for } t < 0$

If A =1, then $r(t) = t^2/2$ and the parabolic function is called unit parabolic function and the corresponding Laplace transform is



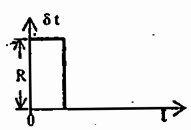
$R(s) = A/s^3$; Parabolic function is also called acceleration function

(4) Impulse function : A unit-impulse is defined as a signal which has zero value everywhere except at $t = 0$, where its magnitude is infinite. It is generally called the δ - function and has the following property : $\delta(t) = 0\ ;\ t \neq 0$

$$\text{Unit impulse function} = \frac{d}{dt}(\text{unit step function})$$

Hence the Laplace transform of unit impulse function is derived from the Laplace transform of unit step function as follows

$\therefore \pounds(\text{unit impulse function}) = s\,.\,1/s = 1$

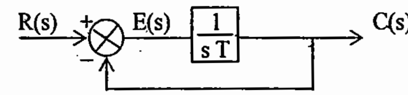


Time response of a First order Control System :

A first order control system is one wherein highest power of 's' in the denominator of its transfer function equals 1. Thus a first order control system is expressed by a transfer function given below :

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

The block diagram representation of the above expression is shown in the below figure.



Block diagram representation of a first order control system.

Time response of a first order control system subjected to unit step input function :

The output for the system is expressed as

$$C(s) = R(s)\frac{1}{sT+1} \longrightarrow (1)$$

As the input is a unit step function $r(t) = 1$ and $R(s) = 1/s$

Therefore, substituting in Eq. (1) $C(s) = \dfrac{1}{s}\cdot\dfrac{1}{sT+1}$

Breaking R.H.S into partial fractions $C(s) = \dfrac{1}{s} - \dfrac{T}{sT+1}$

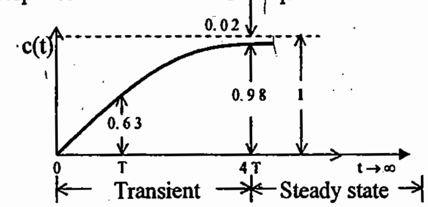
or $C(s) = \dfrac{1}{s} - \dfrac{1}{s+1/T}$ Taking Laplace transform on both sides

$$\pounds^{-1}C(s) = \pounds^{-1}\left[\frac{1}{s} - \frac{1}{s+1/T}\right]\ ;\quad c(t) = 1 - e^{-t/T}$$

The error is given by $e(t) = r(t) - c(t) = 1 - (1 - e^{-t/T}) = e^{-t/T}$

The steady state error $= \lim_{t\to\infty} e^{-t/T} = 0$

The time response in relation to above equation is shown in the figure.



Time response of a first order control system subjected to step input

The graphical representation of the time response shown in figure indicates that the response is exponential type and the steady state value is 1 unit. As the time increases the disparity between the output and input approaches to nil, hence, the steady state error is zero.

Time response of a first order control system subjected to unit ramp input function :

The output for the system is expressed as $C(s) = R(s)\dfrac{1}{sT+1}$

As the input is a unit ramp function is $r(t) = t$ and $R(s) = 1/s^2$

Therefore $C(s) = \dfrac{1}{s^2}\cdot\dfrac{1}{sT+1}$ Breaking R.H.S. into partial fractions

$$C(s) = \frac{1-sT}{s^2} + \frac{T^2}{sT+1}\;;\quad C(s) = \frac{1}{s^2} - T\frac{1}{s} + T\frac{1}{s+1/T}$$

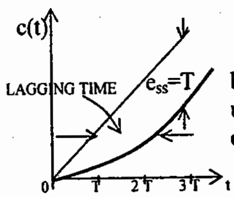
Taking inverse Laplace transform on both sides,

$$\pounds^{-1}C(s) = \pounds^{-1}\left(\frac{1}{s^2} - T\frac{1}{s} + T\frac{1}{s+1/T}\right);\quad c(t) = (t - T + Te^{-t/T})$$

The error is given by $e(t) = r(t) - c(t) = t - (t - T + Te^{-t/T}) = (T - Te^{-t/T})$

The steady state error is $e_{ss} = \lim_{t\to\infty}(T - Te^{-t/T}) = T$

The time response in relation to the above equation is shown in the figure.



The time response shown in figure indicates that during steady state, the output velocity matches with the input velocity but lags behind the input by time T and a positional error of T units exists in the system. It is also observed that lower the time constant lesser is the positional error and also lesser time lag.

Time response of a first order control system subjected to unit impulse input function

The output for the system is expressed as $C(s) = R(s)\dfrac{1}{sT+1}$

As the input to the system is a unit impulse function, its Laplace transform is 1, i.e. R(s) = 1, therefore,

$$C(s) = 1.\frac{1}{sT+1}$$

Taking inverse Laplace transform on both sides of Eq.(2)

$$\pounds^{-1}C(s) = \pounds^{-1}\frac{1}{sT+1} \quad \text{or} \quad \pounds^{-1}C(s) = \pounds^{-1}(1/T).\frac{1}{s+1/T}$$

$$\therefore \quad c(t) = (1/T)e^{-t/T}$$

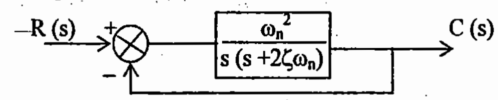
Time Response of Second Order control System :

A second order control system is one wherein the highest power of 's' in the denominator of its transfer function equals 2.

A general expression for the T.F of a second order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The block diagram representation of the transfer function given above is shown in the figure.



Block diagram of a second order control system

Characteristic Equation :

The general expression for the T.F. of a second order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The characteristic equation of a second order control system is given by

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

The location of roots of the chara. equation for various values of ζ (keeping ω_n fixed) and the corresponding time response for a second order control system is shown in below figure.

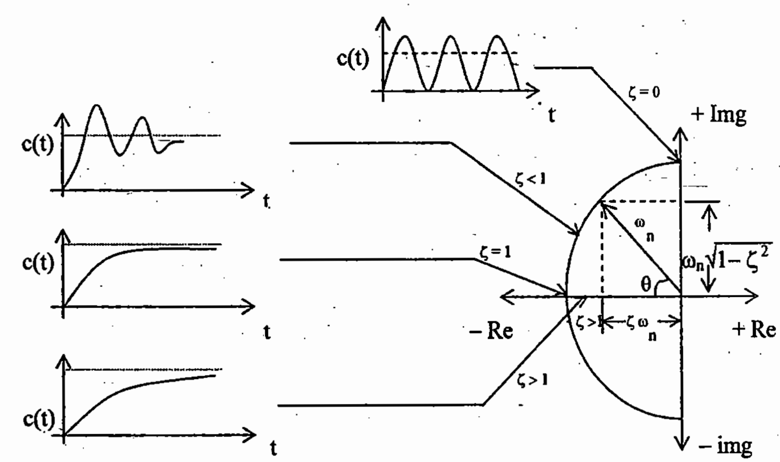


Fig : Location of roots of the characteristic equation and corresponding time response.

From above figure, it is inferred that the change over from underdamped to overdamped Response takes place at $\zeta = 1$. The value of ζ from the location of roots is calculated as

$$\boxed{\zeta = \cos\theta}$$

Time response of a second order control system subjected to unit step input function;

The output for the system is given by

$$C(s) = R(s).\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

As the input is a unit step function $r(t) = 1$ and $R(s) = 1/s$

Therefore, substituting in above Equation

$$C(s) = \frac{1}{s}.\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The solution for the above equation

$$c(t) = 1 - \frac{\exp(-\zeta\omega_n t)}{\sqrt{1-\zeta^2}} \sin\left[(\omega_n\sqrt{1-\zeta^2})t + \left(\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

The time response expression is given by the above equation for values of $\zeta < 1$ is, exponentially decaying oscillations having a frequency $\omega_n \sqrt{(1-\zeta^2)}$ and the time constant of exponential decay is $(1/\zeta\omega_n)$.

Where
- 'ω_n' is called natural frequency of oscillations.
- '$\omega_d = \omega_n \sqrt{(1-\zeta^2)}$' is called damped frequency of oscillations.
- 'ζ' affects the damping and called damping ratio.
- '$\zeta\omega_n$' is called damping factor or actual damping or damping coefficient.

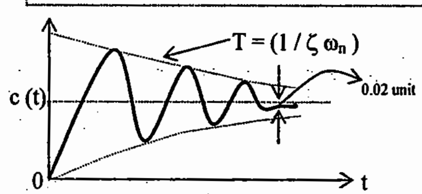


fig. (a)

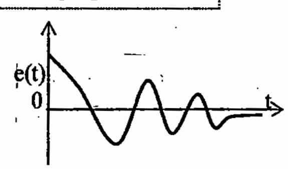


fig. (b)

Fig. Time response and error of a second order control system ($\zeta < 1$, under damped case) subjected to unit step input function.

The time response of a second order control system is influenced by its damping ratio (ζ).The cases for the values of damping ratio as (a) $\zeta < 1$ (b) $\zeta = 0$ (c) $0 < \zeta < 1$ (d) $\zeta = 1$ (d) $\zeta > 1$ are considered below :

UNIT STEP RESPONSE ((O < ζ <1), UNDERDAMPED)

As stated above, if $\zeta < 1$ the time response presents damped oscillation and such a response is called underdamped response.
The response settles within 2% of the desired value (1 unit) after damping out the oscillations in a time *4T*, where $T = (1/\zeta\omega_n)$.

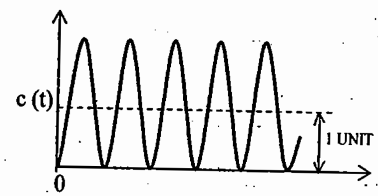
Unit step response when ($\zeta = 0$, undamped system) :

$$c(t) = 1 - \frac{\exp(-0\omega_n t)}{\sqrt{1-0^2}} \sin\left\{\omega_n\sqrt{1-0^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-0^2}}{0}\right)\right\}$$

or $c(t) = 1 - \sin(\omega_n t + \tan^{-1}\infty)$ or $c(t) = 1 - \sin[\omega_n t + \pi/2]$

or $c(t) = (1 - \cos\omega_n t)$

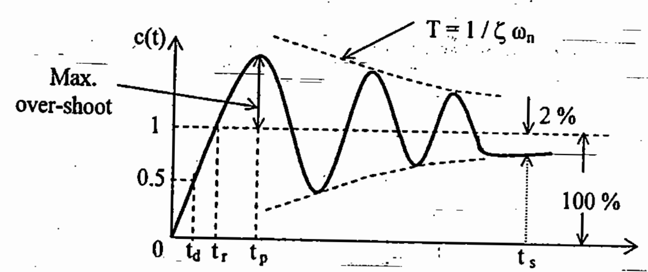
The time response to above equation is plotted in the below figure which indicates sustained (undamped) oscillations.



Unit step response when $0 < \zeta < 1$: (Under Damped systems only)

Transient response specification of second order control system :

The time response of an underdamped control system exhibits damped oscillations prior to reaching steady state. The specifications pertaining to time response during transient part are shown in the following figure.



(1) Delay Time : t_d

The time required for the response to rise from zero to 50% of the final value.

$$t_d = \frac{1 + 0.7\zeta}{\omega_d}$$

(2) The rise time : t_r

The rise time is the time needed for the response to reach from 10 to 90 % or 0 to 100 % of the desired value of the output at the very first instant. Usually 0 – 100 % basis is used for underdamped systems and 10 to 90 % for overdamped system.

$$t_r = \frac{\pi - \phi}{\omega_n \sqrt{1-\zeta^2}} \quad ; \text{ where } \quad \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

(3) peak time : t_P

It is the time required for required for the response to rise zero to peaks of the time response $t_p = \pi / \omega_d$

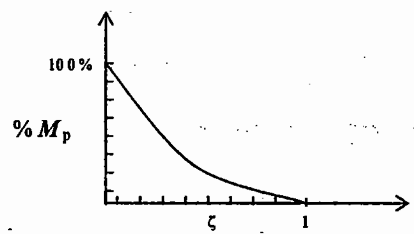
(4) Maximum overshoot : M_P

It gives the normalized difference between time response peak to steady state O/P

$$\text{Percentage } M_P = \frac{c(t)_{max} - C(\infty)}{C(\infty)} \times 100 = \frac{C(t) - 1}{1} \times 100\%$$

$$\% M_p = \exp(-\zeta\pi / \sqrt{1-\zeta^2}) \times 100$$

A graph relating M_p and ζ is plotted in below figure.



Graph between M_p and ζ

(5) The Settling time : t_S

For 2 % tolerance band, the settling order time is given by

$$t_S = 4 \frac{1}{\zeta\omega_n}$$

On 5 % basis the settling time for a second control system is given by

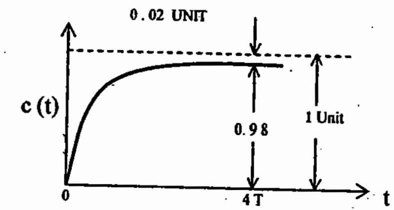
$$t_S = 3 \frac{1}{\zeta\omega_n}$$

An expression for the time response of a second order control system having

$\zeta = 1$ **(critically damped)** when subjected to a unit step input function is:

$$c(t) = [1 - \exp(-\zeta\omega_n t)(1 + \omega_n t)]$$

The time response in relation Eq. (11) is plotted in the below figure. The response is called critically damped response.



Time response of a second order C.S.(ζ =1 , critically damped) subjected to a unit step function.

An expression for the time response of a second order control system having

$\zeta > 1$ **(Overdamped)** when subjected to unit step input function is derived hereunder :

The output for the system is given by

$$C(s) = R(s) \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

As the input is a unit step function $r(t) = 1$ and $R(s) = 1/s$, therefore,

substituting in the above equation $C(s) = (1/s) . \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

It can also be written as $C(s) = (1/s) . \dfrac{\omega_n^2}{(s+\zeta\omega_n)^2 - \omega_n^2(\zeta^2 - 1)}$

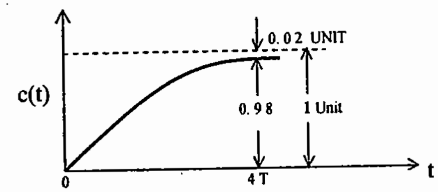
or $C(s) = (1/s) . \dfrac{\omega_n^2}{[s + (\zeta + \sqrt{\zeta^2-1})\,\omega_n]\,[s + (\zeta - \sqrt{\zeta^2-1})\,\omega_n]}$

Expanding R.H.S of above equation into partial fractions,

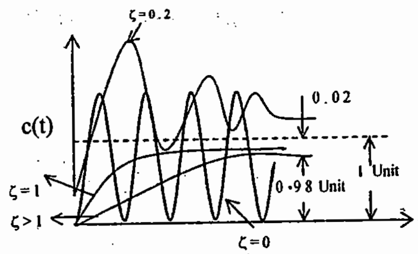
$$C(s) = \frac{1}{s} - \frac{1}{2\sqrt{\zeta^2-1}\,(\zeta - \sqrt{\zeta^2-1})\,[s + (\zeta - \sqrt{\zeta^2-1})\,\omega_n]} + \frac{1}{2\sqrt{\zeta^2-1}\,(\zeta + \sqrt{\zeta^2-1})\,[s + (\zeta + \sqrt{\zeta^2-1})\,\omega_n]}$$

Taking inverse Laplace transform on both sides

$$c(t) = 1 - \frac{\exp[-(\zeta - \sqrt{\zeta^2-1})\,\omega_n t]}{2\sqrt{\zeta^2-1}\,(\zeta - \sqrt{\zeta^2-1})} + \frac{\exp[-(\zeta + \sqrt{\zeta^2-1})\,\omega_n t]}{2\sqrt{\zeta^2-1}\,(\zeta + \sqrt{\zeta^2-1})}$$



Time res. of a second order C.S.($\zeta > 1$, over-damped) subjected to a unit step input function.



Comparison of unit step input time response of a second order control system for different values of 'ζ'.

OBJECTIVE QUESTIONS

01. The radial distance between a pole and the origin gives
   a) damped frequency of oscillation.
   b) undamped frequency of oscillation.
   c) time constant
   d) natural frequency of oscillation.

02. For a type 1, second order control system, when there is an increase of 25 % in its natural frequency, the steady-state error to unit ramp input is
   a) increased by 20 % of its value.
   b) equal to $2\zeta/\omega_n$, where ζ = damping factor.
   c) decreased by 21 %
   d) decreased effectively by 20 %

03. In a type 1, second order system, first peak overshoot occurs at a time equal to

   (a) $\dfrac{\pi\,\omega_n}{\sqrt{1-\zeta^2}}$ (b) $\dfrac{\omega_n}{\sqrt{1-\zeta^2}}$

   (a) $\dfrac{\pi\,\omega_n}{\sqrt{1+\zeta^2}}$ (d) $\dfrac{\pi/\omega_n}{\sqrt{1-\zeta^2}}$

04. Type number of a system gets decreased if
   a) first an integrator and then a differentiator is included in the system.
   b) an integrator is included in the forward path.
   c) a differentiator is included in a parallel path.
   d) a differentiator is included in the forward path.

05. When the pole of a system is moved towards the imaginary axis, then
   a) settling time decreases.
   b) settling time increases by 20 % of initial value.
   c) steady-state error is reduced to zero.
   d) settling time of the system increases.

06. The damping factor of a second order system whose response to unit step input is having sustained oscillations is
   a) = 1 b) > 1
   c) < 1 d) = 0

07. The transient response of a system with feedback when compared to that without feedback
   a) decays slowly.
   b) rises slowly.
   c) rises more quickly.
   d) decays more quickly.

08. The settling time for the system
$$G(s) = \frac{(s+3)}{s^2+5s+25}$$
is .... seconds when the output settles within ± 2 % for a unit step input.
   a) 0.8 b) 1.2
   c) 2.0 d) 1.6

09. The type of the system whose transfer function is given by
$$G(s) = \frac{(s+3)}{s^5+s^4+s^3+3s^2+2s}$$
is
   (a) 3 (b) 2 (c) 5 (d) 1

10. Physically the damping ratio represents the
   a) energy available for transfer.
   b) energy available for exchange.
   c) ratio of energy available for exchange to that available for transfer.
   d) ratio of energy lost to the energy available for exchange.

11. The static acceleration constant of a type 2 system is
   a) infinite b) zero
   c) cannot be found out d) finite.

12. The time domain specification which is dependent only on, the damping factor is
   a) rise time b) peak time
   c) setting time d) peak overshoot.

Key for Objective Questions :

1 to 12 .... (d)

3.2 Steady state Analysis :

The steady state part of time response reveals the accuracy of a control system. Steady state error is observed if the actual output does not exactly match with the input.

$$e(t) = r(t) - c(t)$$

$$\text{Steady state error}, e_{ss} = \underset{t\to\infty}{Lt}\ e(t)$$

$$= \underset{t\to\infty}{Lt}\ [r(t) - c(t)]$$

Using final value theorem,

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,E(s)$$

$$C(s) = E(s)\,G(s) \Rightarrow E(s) = \frac{C(s)}{G(s)}$$

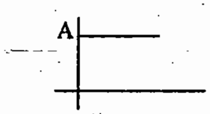
$$\frac{C(s)}{G(s)} = \frac{R(s)}{1+G(s)}$$

$$\therefore\ e_{ss} = \underset{s\to 0}{Lt}\ s\,.\,\frac{R(s)}{1+G(s)}$$

The open-loop transfer function, the type indicates the number of poles at the origin and the order indicates the total number of poles. The type of the system determines steady state response and the order of the system determines transient response.

Standard test signals used in Steady state response :

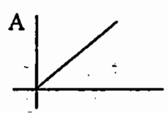
(1) Step input signal :



$$r(t) = A\,u(t)$$

$$R(s) = A/s$$

(2) Ramp input signal :



$$r(t) = A\,t$$

$$R(s) = A/s^2$$

(3) Parabolic input signal :

$$r(t) = A\,t^2/2$$

$$R(s) = A/s^3$$

For step input

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,.\,\frac{R(s)}{1+G(s)}$$

$$= \underset{s\to 0}{Lt}\ s\,.\,\frac{A/s}{1+G(s)} = \underset{s\to 0}{Lt}\ \frac{A}{1+G(s)} = \frac{A}{1+K_p}$$

where $K_p = \underset{s\to 0}{Lt}\ G(s)$ = Position error constant

For ramp input

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,.\,\frac{R(s)}{1+G(s)}$$

$$= \underset{s\to 0}{Lt}\ s\,.\,\frac{A/s^2}{1+G(s)} = \underset{s\to 0}{Lt}\ \frac{A}{s\,[1+G(s)]} = \frac{A}{K_v}$$

where $K_v = \underset{s\to 0}{Lt}\ s\,G(s)$ = Velocity error constant

For parabolic input

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,.\,\frac{R(s)}{1+G(s)}$$

$$= \underset{s\to 0}{Lt}\ s\,.\,\frac{A/s^3}{1+G(s)} = \underset{s\to 0}{Lt}\ \frac{A}{s^2\,[1+G(s)]} = \frac{A}{K_a}$$

where $K_a = \underset{s\to 0}{Lt}\ s^2\,G(s)$ = Acceleration error constant

	Type 0	1	2	3
Step	$\frac{A}{1+K_p}$	0	0	0
Ramp	∞	A/K_v	0	0
Parabolic	∞	∞	A/K_a	0

OBJECTIVE QUESTIONS

01. The presence of non-linearities in a control system tends to introduce
a) transient error b) instability
c) static error d) steady-state error

02. The static acceleration constant of a type 2 system is
a) infinite b) zero
c) cannot be found out d) finite

03. The transfer function of the system which will have more steady state error for step input is
(a) $\frac{80}{(s+1)(s+2)(s+3)}$
(b) $\frac{120}{s(s+1)(s+15)}$
(c) $\frac{60}{(s+0.5)(s+3)(s+5.5)}$
(d) $\frac{120}{(s+1)(s+4)(s+15)}$

04. The presence or absence of steady-state error for any given system depends upon
a) presence or absence of pole at the infinity.
b) presence or absence of poles and zeros at the origin.
c) absence or presence of zeros at the origin.
d) absence or presence of pole at the origin.

05. When the gain 'k' of a system is increased, the steady-state error of the system
a) increases.
b) remains unchanged
c) may increase or decrease.
d) decreases.

06. The plant is represented by the transfer function.The system is given a degenerative feedback. The effective of the feedback is to shift the pole
a) positively to $s = (a + k)$ and reduce the time constant to $\alpha + (1/k)$
b) negatively to $s = -(\alpha + k)$ and increase the time constant to $\alpha + k$.
(c) negatively to $s = -(\alpha + k)$ and reduce the time constant to $\alpha + (1/k)$.
(d) negatively to $s = -(\alpha + k)$ and decrease the time constant to $[1/(\alpha+k)]$.

07. In a system with input R (s) and output C (s), the transfer functions of the plant and the feedback system is given by G (s) and H (s) respectively. The system has got a negative feedback. Then the error signal is given by the expression :
(a) $E(s) = \frac{G(s)R(s)}{1+G(s)H(s)}$
(b) $E(s) = C(s)G(s)$
(c) $E(s) = \frac{1}{1+G(s)H(s)}$
(d) $E(s) = \frac{R(s)}{1+G(s)H(s)}$

08. The static error constants depends on
a) the order of the system
b) the type of the system
c) both type and order of the system
d) None of the above

Key :

1. d 2. d 3. d 4. d 5. d
6. d 7. d 8. b 9. d

JTO Previous Questions

01. Control systems are normally designed with damping factor
(A) more than unity
(B) of zero
(C) less than unity
(D) of unity

02. A type – 2 system has a finite non – zero value of error constant
(A) ∝ (B) zero
(C) K_a (D) $1/K_a$

03. The type 2 system has which of the following
(A) constant position error as well as velocity error
(B) zero position error and constant velocity error
(C) constant position error and zero velocity error
(D) zero position error as well as velocity error

04. The time required for the response to reach half the final value for the first time is
(A) decay time (B) rise time
(C) pick – up time (D) delay time

05. For the characteristic equation $S^2 + 4.8s + 72 = 0$, the damping ratio and natural frequency respectively are?
(A) 0.812, 9.1 rad/sec
(B) 0.283, 8.48 rad/sec
(C) 0.256, 8.31 rad/sec
(D) 0.913, 8.5 rad/sec

06. The type – 0 system has a finite non – zero value of
(A) K_V (B) K_a
(C) K_p (D) all of the above

07. A unity feed back system has transfer function $G(s) = \frac{9}{S(S+3)}$, its?
(A) natural frequency = 2
(B) natural frequency = 3
(C) damping ratio = 0.5
(D) damping ratio = 0.9

08. The system whose characteristic equation $s^2 + 6s + 5 = 0$ has the following roots
(A) – 3, - 2, - 1 (B) – j, j, - 1, 1
(C) – 3, - 2, 0 (D) – 2+3j, -2-3j-2

09. The overshoot of the system having the transfer function $25/s^2+25$ For a unit step input applied would be
(A) 20% (B) 30%
(C) 35% (D) 100%

10. Potentiometers are used in control system
(A) to improve stability
(B) to improve frequency response
(C) as error sensing transducer
(D) to improve time response

11. The position and velocity errors of a type 2 system are
(A) zero, constant
(B) constant, constant
(C) zero, zero
(D) constant, infinity

12. Bandwidth is used as means of specifying performance of a control system related to
(A) the speed of response
(B) the constant gain
(C) relative stability of the system
(D) all of the above

13. The value of steady – state error to the type 1 system, when the input signal is a step of magnitude 2, will be

(A) 0.5 (B) 2.5
(C) 1.54 (D) zero

14. Which of the following system is generally preferred

(A) critically damped
(B) under damped
(C) over damped
(D) oscillatory

15. As the system type becomes higher steady state error
(A) remains constant
(B) increases
(C) is eliminated
(D) none of the above

16. The steady state acceleration error for a type 1 system is

(A) between zero and unity
(B) zero
(C) infinite
(D) unity

17. For a second order differential equation of the damping ratio is 1, them
(A) the poles are equal, negative and real
(B) both poles are negative and real
(C) the poles are in right half of these plane
(D) the poles are in a left half of the s plane

18. The position and velocity errors of a type 2 systems

(A) zero, infinity
(B) zero, zero
(C) zero, constant
(D) constant, zero

19. With the feedback system, the transient response

(A) rises slowly
(B) rises quickly
(C) decays slowly
(D) decays rapidly

20. Error constants of a system are measure of

(A) steady state response
(B) steady state as well as transient state response
(C) relative stability
(D) transient state response

21. Static error coefficients are used as a measure of the effectiveness of closed – loop systems for specified

(A) velocity input signal
(B) acceleration input signal
(C) position input signal
(D) all of the above

22. The transient response of a system is mainly due to

(A) friction (B) stored energy
(C) internal forces (D) inertia forces

23. For any given closed loop system
(A) all the coefficients are always non= zero
(B) all the coefficients can have zero value
(C) only one of the static error coefficients has a finite non zero value
(D) any of the above

24. A unity feedback system has open – loop transfer function $G(s) = 1/(1+s)$. The pole of the closed loop system is located on the real axis in the S – plane is

(A) – 2 (B) – 1
(C) 2 (D) – 5

25. The output of a linear system for a unit step input is given by t^2e^{-1t} The transfer function is given by

(A) $\frac{2s}{(s+1)^3}$ (B) $\frac{2}{s(s+1)^2}$

(C) $\frac{1}{s^2(s+1)}$ (D) $\frac{s}{(s+1)^3}$

26. The steady – state error coefficient for a system are given by $K_p = 0$, K_V = finite constant, $K_a = \infty$. The system is

(A) Type – 1 system
(B) Type – 2 system
(C) Type – 3 system
(D) Type – 0 system

27. In the case of second order differential equation when the damping ratio is less than 1 then
(A) poles will be unequal
(B) poles will be complex conjugate & negative
(C) poles will be equal, negative and real
(D) poles will be positive

28. The error signal produced in a control system is a constant. The output of P action will be

(A) linear (B) infinity
(C) constant (D) zero

29. The transient response of the system depends on

(A) input (B) output
(C) system (D) none

30. The steady – state response of the system depends on

(A) input (B) output
(C) system (D) Input & output

31. The normal range of damping ratio for a control system is

(A) 0.5 to 1.0 (B) 0.8 to 2.0
(C) 0.3 to 0.5 (D) 0.5 to 2.5

Previous PSU's Question

01. The steady state error due to a ramp input for a type two system is equal to

(A) zero (B) infinite
(C) constant (D) data is insufficient

02. Given the transfer function

$$G_3(s) = \frac{121}{s^2 + 13.2\,s + 121}$$

of a system. Which of the following characteristics does it have?

(A) Overdamped and settling time 1.1s.
(B) Underdamepd and settling time 0.6 s.
(C) Critically damped and settling time 0.8 s
(D) uncerdamped and settling time 0.707s.

03. Consider the following statements with a reference to a system with velocity error constant $k_v = 1000$

1. The system is stable
2. The system is of type 1
3. The test signal used is a step input.

Which of these statements are correct?

(A) 1 & 2 (B) 1 & 3
(C) 2 & 3 (D) 1, 2, & 3

04. The close loop transfer function of control system is given

$$\frac{C(s)}{R(s)} = \frac{1}{1+s}$$

For the input r(t) = -sint, the steady state value of C(t) is equal to

(A) $\frac{1}{\sqrt{2}}$ Cos t (B) 1

(C) $\frac{1}{\sqrt{2}}$ Sin t (D) $\frac{1}{\sqrt{2}}$ Sin $(t - \pi/4)$

05. The transient response of a system is mainly due to

(A) internal forces
(B) Stored energy
(C) Friction
(D) inertia forces

06. A system is critically damped. Now if the gain of the system is increased. The system will behaves as

(A) over damped
(B) Underdamprd
(C) oscillatory
(D) Critically damped

07. Consider a system with the transfer function

$$G(s) = \frac{S+6}{Ks^2+s+6}$$

It damping ratio will be 0.5 when the value of k is

(A) 2/6 (B) 3 (C) 1/6 (D) 6

08. The transfer function of a control system is given as

$$\frac{k}{s^2+4s+k)}$$

Where k is gain of the system in radian/ amp. For this system to be critically damped the value of k should be

(A) 1 (B) 2 (C) 3 (D) 4

09. The unit impulse response of a system is given by $c(t) = 0.5e^{-t/2}$. Its transfer function is

(A) $\frac{1}{(s+2)}$ (B) $\frac{1}{(1+2s)}$

(C) $\frac{2}{(1+2s)}$ (D) $\frac{1}{(s+2)}$

10. The impulse response of a system is given by $C(t) = ½ e^{-t/2}$. Which one of the following is its unit step responses.

(A) $1 - e^{-1/2t}$ (B) $1 - e^{-t}$
(C) $2 - e^{-2t}$ (D) $1 - e^{-2t}$

11. A system is represented by

$$\frac{dy}{dt} + 2y = 4t\,u(t)$$

The ramp component in the forced response will be

(A) t u(t) (B) 2t u(t)
(C) 3t u(t) (D) 4t u (t)

12. Which one of the following is the steady state error of a step input applied to a unity feedback will e the open loop Transfer function

$$G(s) = \frac{10}{s^2+14+50}$$

(A) $e_{ss} = 0$ (B) $e_{ss} = 0.83$
(C) $e_{ss} = 1$ (D) $e_{ss} = \infty$

13. The unit step response of a particular control system is given by $c(t) = 1 - 10e^{t}$, then its transfer function is

(A) $\frac{10}{s+1}$ (B) $\frac{s-9}{s+1}$

(C) $\frac{1-9s}{s+1}$ (D) $\frac{1-9s}{s(s+1)}$

14. The open loop transfer function

$$G(s) \text{ of } \frac{1}{s(s+1)}$$

an unity feed back control system is The system is subjected to an input r(t) sin t. The steady state error will be

(A) zero (not defined)
(B) 1
(C) $\sqrt{2}\sin(t - \pi/4)$
(D) $\sqrt{2}\sin(t + \pi/4)$

15. A second order system has the damping ratio ξ and undamped natural frequency of oscillation ω_n, the settling time at 2% tolerance band of the system is

(A) $\frac{2}{\xi\omega_n}$ (B) $\frac{3}{\xi\omega_n}$

(C) $\frac{4}{\xi\omega_n}$ (D) $\xi\omega_n$

16. The response c(t) to a system is described by the differential equation

$$\frac{d^2c(t)}{dt^2} + \frac{4dc(t)}{dt} + 5\,c(t) = 0;$$

The system response is

(A) undamped
(B) underdamped
(C) critically damped
(D) oscillatory

17. The steady state error of a stable type unity feed back system for a unit step function is

(A) 0 (B) $\frac{1}{1+K_p}$

(C) ∞ (D) $\frac{1}{K_p}$

18. What is the steady state error for a unity feedback control system having

$$G(s) = \frac{1}{s(s+1)}$$

due to unit ramp input?

(A) 1 (B) 0.5
(C) 0.25 (D) √0.5

19. When the time period of observation is large, the type of the error is

(A) Transient error
(B) steady state error
(C) Half – power error
(D) position error constant

20. For which of the following input, the error series using dynamic error coefficients doesn't converge

(A) step input
(B) ramp input
(C) acceleration (parabolic) input
(D) sinusoidal input

JTO KEYS:

01. C	02. C	03. D	04. D	05. B
06. C	07. B	08. C	09. D	10. C
11. C	12. A	13. B	14. B	15. C
16. B	17. A	18. B	19. D	20. A
21. D	22. B	23. C	24. A	25. A
26. A	27. B	28. C	29. C	30. A
31. A				

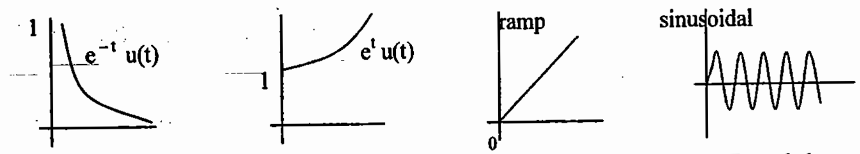
PSU's KEYS:

01. A	02. B	03. A	04. D	05. B
06. B	07. C	08. D	09. B	10. A
11. B	12. B	13. C	14. A	15. C
16. B	17. B	18. A	19. B	20. D

Concept of stability:

Any system is called as a *stable system* if the output of the system is bounded for a bounded input. Any signal is called bounded if the max. and min. value are finite.

Eg :



Bounded   Unbounded   Unbounded   Bounded

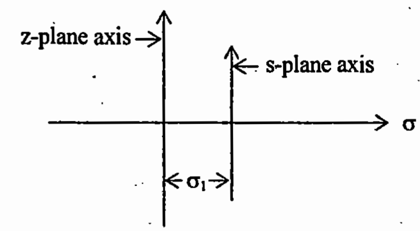
* Stability of any system depends only on the location of poles but not on the location of zeros.
* If the poles are located in left side of s – plane, then the system is stable.
* If the roots are located on imaginary axis including the origin (except repeated roots), the system is stable.
* If the poles are located in right half of s – plane, then the system is unstable.
* As pole is approaches origin, stability decreases.
* When roots are located on imaginary axis, then the system is marginally stable.
* The poles which are close to the origin are called dominant poles.
* The systems are classified as
  1) Absolutely stable systems
  2) Unstable systems
  3) Conditionally stable systems
* When variable parameter is varied from 0 to ∞, if the poles are located on left side and it is always stable, then it is absolutely stable.
* When variable parameter is varied and a system is stable for values 0 to ∞, at some point onwards there is (are) pole(s) in right side then it is called conditionally stable.
* Techniques used to calculate stability are
  1) Routh-Hurwitz criterion
  2) Root locus
  3) Bode plot
  4) Nyquist plot
  5) Nicholas chart

Relative Stability Analysis :

Once a system is shown to be stable, we proceed to determine its relative stability quantitatively by finding the settling time of the dominant roots of its characteristic equation. The settling time is inversely proportional to the real part of the dominant roots, the relative stability can be specified by requiring that all the roots of the characteristic equation be more negative than a certain value, i.e., all the roots must lie to the left of the lines $s = -s_1 (s_1 > 0)$. The characteristic equation of the system under study is then modified by shifting the origin of the s-plane to $s = -\sigma_1$, i.e., by the substitution

$$s = z - s_1$$

If the new characteristic equation in z satisfies the Routh criterion, it implies that all the roots of the original characteristic equation are more negative than $-\sigma_1$.



4.1 ROUTH – HURWITZ CRITERION :

The Routh – Hurwitz criterion represents a method of determining the location of poles of a polynomial with constant real coefficients with respect to the left half and the right half of the s-plane, without actually solving for the poles.

Consider that the characteristic equation of a linear time-invariant system is of the form

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots\dots + a_{n-1} s + a_n = 0$$

In order that there be no roots of the last equation with positive real parts, it is necessary but not sufficient that

1. All the coefficients of the polynomial have the same sign.
2. None of the coefficients vanishes.

The necessary and sufficient condition that all roots of above equation lies in the left half of the s-plane is that the Polynomial's Hurwitz determinants must all be positive.

The roots of the polynomials are all in the left half of the s-plane if all the elements of the first column of the Routh tabulation are of the same sign. If there are changes of signs in the elements of the first column, the number of sign changes indicates the number of roots with positive real parts.

The following difficulties may occur occasionally when carrying out the Routh test :

1. The first element in any one row of the Routh tabulation is zero, but the other elements are not.
2. The elements in one row of the Routh tabulation are all zero.

Example : Consider the following equation :

$$s^5 + 1.5\,s^4 + 2\,s^3 + 4\,s^2 + 5\,s + 10 = 0$$

The Routh array is given below :

s^5	1	2	5
s^4	1.5	4	10
Sign change			
s^3	–0.66	–0.66	0
Sign change			
s^2	0.227	10	0
s^1	27.4	0	0
s^0	10	0	0

As there are two sign changes in first column of Routh table, it is concluded that the system under consideration is unstable having two poles in the right half of the s-plane.

Example : Consider the characteristic equation :

$$s^4 + 5\,s^3 + 5\,s^2 + 4\,s + K = 0$$

The Routh array for this equation is

s^4	1	5	K
s^3	5	4	12
s^2	21/5	K	
s^1	[84/5 – 5K] / (21/5)		
s^0	K		

Since for a stable system, the signs of elements of the first column of the Routh array should be all positive, the condition of system stability requires that

$$K > 0$$

and $\quad (84/5 - 5K) > 0$

Therefore for stability, K should be lie in the range

$$84/25 > K > 0$$

Special Cases :

***Difficulty* 1 :** *When the first term is in any row of the Routh array is zero while rest of the row has at least one nonzero term.*

Because of this zero term, the terms in the next row become infinite and Routh's test breaks down. The following method can be used to overcome this difficulty.

Example : Determine the stability of a closed-loop control system whose characteristic equation is

$$s^5 + s^4 + 2\,s^3 + 2\,s^2 + 11\,s + 10 = 0$$

The Routh array is formed below :

s^5	1	2	11
s^4	1	2	10
s^3	0	1	0
s^2			

while forming the Routh array as above, the third element in the first column is zero and thus the Routh criterion fails at this stage. The difficulty is solved if zero in the third row of the first column is replaced by a symbol 'ε' and Routh array is formed as follow :

s^5	1	2	11
s^4	1	2	10
s^3	ε	1	0
s^2	$\lim_{\varepsilon \to 0} \left(\frac{2\varepsilon - 1}{\varepsilon}\right) = 0$	10	0
s^1	$\lim_{\varepsilon \to 0} \left(1 - \frac{10\,\varepsilon^2}{2\varepsilon - 1}\right) = 1$	0	0
s^0	10		

The limits of the fourth and fifth element in the first column as $\varepsilon \to 0$ from positive side are $-\infty$ and +1 respectively indicating two sign changes, therefore, the system is unstable and the number of roots with positive real parts of the characteristic equation is 2.

***Difficulty* 2 :** *When all the elements in any one row of the Routh array are zero.*

This condition indicates there are symmetrically located roots in the s-plane. Because of a zero row in the array, the Routh's test breaks down. This situation is overcome by replacing the row of zeros in the Routh array by a row of coefficients of the polynomial generated by taking the first derivative of the auxiliary polynomial. The following example illustrates the procedure.

Example : Determine the stability of a system having following characteristic equation :

$$s^6 + s^5 + 5\,s^4 + 3\,s^3 + 2\,s^2 - 4\,s - 8 = 0$$

The Routh table is formed below :

s^6	1	5	2	– 8	
s^5	1	3	– 4	0	
s^4	2	6	– 8	0	Auxiliary equation
s^3	0	0	0	0	

It is observed in this example that all the elements in the fourth row vanish and the application of Routh criterion falls. This situation occurs when the array has two consecutive rows having the same ratio of corresponding elements.

This difficulty faced is overcome by forming an auxiliary equation using elements of the last but one vanishing row. The derivative of this auxiliary equation is taken w.r.t. 's' and the coefficients of the differentiated equation are taken as the elements of the following row :

The auxiliary equation is

$$A(s) = 2s^4 + 6s^2 - 8$$

And $dA(s)/ds = 8s^3 + 12s - 0$

The coefficients of the fourth row are thus 8, 12 and 0. The modified Routh array is given below :

s^6	1	5	2	−8
s^5	1	3	−4	0
s^4	2	6	−8	0
s^3	8	12	0	0
s^2	3	−8	0	0

coefficients of differentiated A.E.

s^1	100 / 3	0	0	0

Sign change

s^0	−8	0	0	0

As there is one sign change in the first column, the system has one root with positive real part indicating that the system is unstable.

OBJECTIVE QUESTIONS

Q01. The transfer function of a system is

$$\frac{K}{a_3 s^3 + a_2 s^2 + a_1 s + a_o}$$

. For the system to be absolutely stable,

a) $a_3, a_2, a_1, a_o > 0$ and $a_2 a_1 - a_3 a_o > 0$
b) $a_3, a_2, a_1, a_o > 0$ and $a_2 a_1 - a_3 a_o < 0$
c) $a_3, a_2, a_1, a_o > 0$ and $a_2 a_1 - a_3 a_o = 0$
d) $a_2, a_o > 0$ and $a_3, a_1 < 0$

02. Routh's array for a system is giver below.

s^4	1	3	5
s^3	1	2	0
s^2	1	5	
s^1	−3		
s^o	5		

The system is
a) stable
b) unstable
c) marginally stable
d) conditionally stable

03. The number of sign changes in the entries in the first column of Routh's array denotes

a) the number of zeros of the system in the RHP
b) the number of roots of characteristic polynomial in RHP
c) the number of open-loop poles in RHP
d) the number of open-loop zeros in RHP

04. Consider a characteristic equation $s^4 + 3s^3 + 5s^2 + 6s + K + 10 = 0$. The condition for stability is

a) $K > 5$ b) $-10 \le K$
c) $K > -4$ d) $-10 < K < -4$

05. The characteristic equation of a unity feedback system is given by

$$s^3 + s^2 + 4s + 4 = 0$$

a) The system has one pole in the RH- *s* plane
b) The system has no poles in the RH – *s* plane
c) The system is asymptotically stable
d) The system exhibits oscillatory behaviour

06. An electromechanical closed-loop control system has the following characteristic equation $s^3 + 6Ks^2 + (K+2)s + 8 = 0$ where K is the forward gain of the system. The condition for closed-loop stability is

a) K = 0.528 b) K = 2
c) K = 0 d) K = − 2.528

07. The case in the Routh table in which a particular row elements are zero, show that
a) a differentiation has to be carried out and conveys no other information
b) it is a special case in Routh array
c) whether the system is stable or not
d) some roots are distributed symmetrically about the origin.

08. The system is represented by its transfer function has some poles lying on the imaginary axis, it is
a) unconditionally stable
b) conditionally stable
c) unstable
d) marginally stable

09. In the first column of the Routh array, an element was found to be zero. The first column element above this zero and below this zero has the same sign. This condition indicate that the system
a) is stable b) is unstable
c) has all the roots in LHP except one
d) has some roots on the jω-axis

10. The statements that holds good for relative stability analysis is :
   a) Routh array method cannot be used
   b) graphical methods can be used and Routh array cannot be used
   c) graphical methods cannot be used
   d) both graphical as well as Routh array can be used

Key for Objective Questions :

1. a 2. b 3. b 4. d 5. b, d
6. a, d 7. d 8. d 9. d 10. d

JTO Previous Question

01. The closed loop transfer function of system is given by
   $C(s)/R(s) = 1/[(s+2)(s^2+4)]$.
   The system is
   (A) completely unstable
   (B) completely stable
   (C) marginally stable
   (D) conditionally stable

02. A system which has some roots with real parts equal to zero, but none with +ve real parts, is
   (A) absolutely unstable
   (B) absolutely stable
   (C) relatively stable
   (D) marginally stable

03. A step function is applied to the input of a system and output is of the form $y = t$, the system is
   (A) conditionally stable
   (B) not necessarily stable
   (C) un stable
   (D) stable

04. When gain K of the loop transfer function is varied from zero to infinity the closed loop system
   (A) stability is improved
   (B) always become unstable
   (C) stability is not affected
   (D) may become unstable

05. In a non – linear control system limit cycle is self sustained oscillations of
   (A) variable amplitude
   (B) fixed frequency and amplitude
   (C) fixed frequency
   (D) variable frequency

06. In a control system, the use of negative feedback
   (A) increases the influence of variations of component parameters on the system performance
   (B) reduces the effects of disturbance and noise signals in the forward path
   (C) increases the reliability
   (D) eliminates the chances of instability

07. If a step function is applied to the input of a system and the output remains below a certain level for all the time, the system is
   (A) stable (B) marginally stable
   (C) not necessarily stable (D) unstable

08. The condition that all roots of the following polynomial
   $a_0s^3 + a_1s^2 + a_2 s + a_3 = 0$
   (A) $a_1a_2 > a_0a_3$ (B) $a_2a_0 > a_1a_3$
   (C) $a_1a_3 > a_0a_2$ (D) $a_1a_2 > a_0a_3$

09. Which of the following statements is in – corrent?
   (A) If the output response to a bounded input signal results in constant amplitude or constant amplitude oscillation, then the system is limited stable
   (B) If a system response is stable for all variation of its parameters, it is called absolutely stable system
   (B) If a system response is stable for a limited range of variations of its parameters, it is called conditionally stable system
   (D) If a system output is an oscillatory signal for a sinusoidal signal, it is called relatively stable

10. If a system is subjected to an unbounded input produce an unbounded response, then the system is
   (A) nothing can be predicted about its stability
   (B) absolutely stable
   (C) unstable
   (D) stable

KEYS :

(1) C (2) D (3) C (4) D (5) B
(6) B (7) C (8) B (9) D (10) A

4.2 ROOT LOCUS TECHNIQUE

It is the graphical representation of the roots of the characteristic equation, then the variable parameter is varied from 0 to ∞.

1) Root Locus (RL)	($K \to 0$ to ∞)
2) Complementary RL	($K \to 0$ to $-\infty$)
3) Complete RL	($K \to -\infty$ to ∞)
4) Root contour	(Multiple parameter variation)

Concept of Root locus :

It is not possible to plot the root locus if there is no variable parameter in characteristic equation.

Classification of stable systems :

1) Undamped system	(roots on imaginary axis i.e., real part = 0)
2) Under damped system	(imaginary but real part is negative)
3) Critically damped	(roots are real and same)
4) Over damped system	(roots are real and different)

Rules for the construction of Root locus :

* The root locus is always symmetrical with respect to the real axis.
* The root locus always starts (K= 0) from the open - loop poles and terminates (K= ∞) on either finite open – loop zeros or infinity. This statement is valid only if P = Z.
* The number of separate branches of the root locus equals either the number of open – loop poles or number of open – loop zeros whichever is greater.

$N = P$, if $P > Z$
$N = Z$, if $Z > P$

* A section of root locus lies on the real axis if the total number of open – loop poles and zeros to the right of the section is odd.
* The value of 'K' at any point on the root locus can be calculated by using the magnitude criteria.

$$K = \frac{\text{Product of poles magnitude (or length)}}{\text{Product of zeros magnitude (or length)}}$$

* If $P \neq Z$, some of the branches terminate at '∞' or some of the branches will start from '∞'.

If $P > Z$, $(P - Z)$ branches will terminate at '∞'.
If $Z > P$, $(Z - P)$ branches will start from '∞'.

→ Whenever any branch will terminate at '∞' means that a zero is located at '∞'.
→ Whenever any branch is start from '∞' means that a pole is located at '∞'.

* **The angle of asymptotes :**

If $P > Z$, $(P - Z)$ branches will terminate at '∞' along straight line asymptotes whose angles are

$$\frac{(2q+1)\,180^\circ}{P-Z}$$

If $Z > P$, $(Z - P)$ branches will start from '∞' along straight line asymptotes whose angles are

$$\frac{(2q+1)\,180^\circ}{Z-P}$$

Centroid : The asymptotes meet the real axis at centroid.

$$\sigma = \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of zeros}}{P - Z}$$

* **Intersection points with imaginary axis :** The value of 'K' and the point at which the Root locus branch crosses the imaginary axis is determined by applying Routh criterion to the characteristic equation. The roots at the intersection point are imaginary.
* **Break – away point and break – in point :**

Break – away point is calculated when root locus lies between two poles.
Break – in point is calculated when root locus lies between two zeros.

Break – away or break – in point is calculated by solving $\boxed{\dfrac{dK}{ds} = 0}$

Procedure :

a) From the characteristic equation (C.E.)
b) Rewrite the characteristic equation in the form of $K = f(s)$
c) $dK / ds = 0$
d) The root of $dk / ds = 0$ gives the valid and invalid break point
c) The valid break point which must be on root locus branch

* **Angle of arrival :**

It is applied when there are complex zeros.

$$\boxed{\phi_A = 180^\circ + \phi}$$

where $\phi = \angle\text{poles} - \angle\text{zeros}$

* **Angle of departure :**

It is applied when there are complex poles.

$$\boxed{\phi_D = 180^\circ - \phi}$$

Complementary Root locus :

In this the magnitude criteria remains same but angle criteria changes.

i.e., $\angle$Zeros – $\angle$ Poles = even multiples of π

1) Asymptotic angles $= \dfrac{(2q)\,180^\circ}{P-Z}$

2) Angle of departure $= 180^\circ - \phi$

where $\phi = \angle$poles – $\angle$ zeros

3) Angle of arrival $= 180^\circ + \phi$

4) A point on the real axis lies in the complementary RL, if the number of poles and zeros to the right side of any point is a even number.

Example : Sketch the complete root locus for the system having

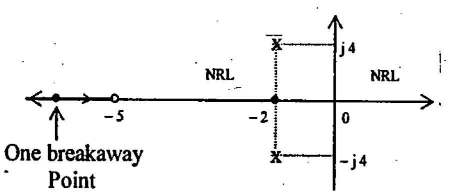
$$G(s)\,H(s) = \frac{K\,(s+5)}{(s^2+4s+20)}$$

Sol : *Step 1* : Number of poles $P=2$, $Z=1$, $N=P-Z$

One branch has to terminate at finite zero $s=-5$ while $P-Z=1$ branch has to terminate at ∞.

Starting points of branches are, $-2 \pm j\,4$.

Step 2 : Pole-zero plot of the system is shown below.



Step 3 : Angle of asymptotes

$$\theta = \frac{(2q+1)\,180^0}{P-Z}, \qquad q=0$$

Step 4 : Centroid.

As there is one branch approaching to ∞ and one asymptote exists, centroid is not required.

Step 5 : Breakaway point.

$$1 + G(s)\,H(s) = 0$$

$$\therefore \quad s^2 + 4s + 20 + K\,(s+5) = 0$$

$$\frac{dK}{ds} = 0 \qquad \Rightarrow -s\,(s+10) = 0$$

$s=0$ and $s=-10$ are breakaway points. But $s=0$ cannot be breakaway point. Hence $s=-10$ is valid breakaway point.

Step 6 : Intersection with imaginary axis.

Characteristic equation ,

$$s^2 + 4s + 20 + K\,(s+5) = 0$$

$$s^2 + s\,(K+4) + (20+5K) = 0$$

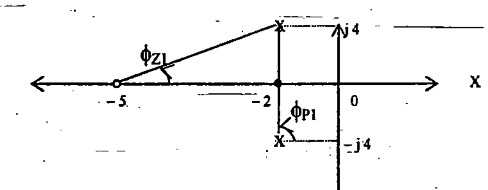
Routh's Array can be formed as below :

s^2	1	$20+5K$
s^1	$K+4$	0
s^0	$20+5K$	

$K_{mar} = -4$ makes s^1 row as row of zeros.

But as it is negative, there is no intersection of root locus with imaginary axis.

Step 7 : Angle of departure.

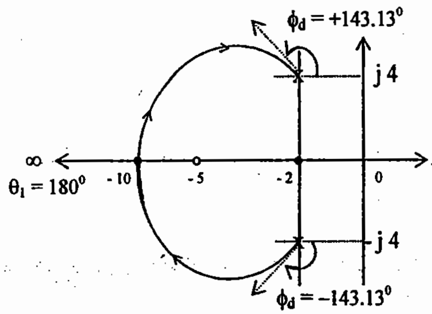


$\phi_{P1} = 90^0$, $\qquad \phi_{Z1} = \tan^{-1}(4/3) = 53.13^0$

$\therefore \quad \phi = \sum\phi_P - \sum\phi_Z = 36.86^0$

$\therefore \quad \phi_d = 180^0 - \phi = +143.13^0$ at $-2+j\,4$ pole.

$\phi_d = -143.13^0$ at $-2-j\,4$ pole.



Step 9 : Prediction of stability

For all ranges of K i.e., $0 < K < \infty$. both the roots are always in left half of s-plane. So system is inherently stable.

Example : Sketch the complete root locus of system having

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$$

Sol. : *Step 1* : P = 4, Z = 0 & N = 4 i.e., four branches in the root locus.

Step 2 : All four branches starts from open-loop poles and terminates at ∞.

Step 3 : Angle of asymptotes $= \dfrac{(2q+1)\,180^0}{4} = 45^0, 135^0, 225^0, 315^0$

Step 4 : Centroid $= \dfrac{0-1-2-3}{4} = -1.5$

Step 5 : Breakaway point

$$K = -s^4 - 6s^3 - 11s^2 - 6s$$

$$\frac{dK}{ds} = 0 \Rightarrow s = -1.5, -0.381, -2.619$$

Here, – 1.5 lies in the root locus and – 0.381, & 2.619 lies in the complementary root locus.

Step 6 : Intersection of root locus imaginary axis.

Characteristic Equation $s^4 + 6s^3 + 11s^2 + 6s + K = 0$

s^4	1	11	K
s^3	6	6	0
s^2	10	K	0
s^1	(60 – 6K) / 10	0	
s^0	K		

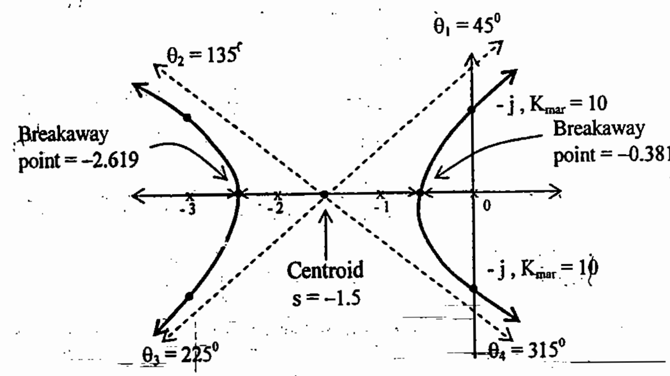
$\therefore \quad 60 - 6K = 0 \qquad \therefore K_{mar} = +10$

Auxiliary equation :

$$10s^2 + K = 0$$

At K = 10, $s^2 = -1$, $s = \pm j$

Step 7 : Complete root locus.



Step 8 : For 0 < K < 10, system is absolutely stable. At K = 10, system is marginally stable oscillating with 1 rad/sec. For K > 10, system is unstable.

Complementary Root Locus :

Step 1 : P = 4, Z = 0 & N = 4 i.e., four branches in the root locus.

Step 2 : All four branches starts from open-loop poles and terminates at ∞.

Step 3 : Angle of asymptotes $= \dfrac{(2q)\,180^0}{4} = 0^0, 90^0, 180^0, 270^0$

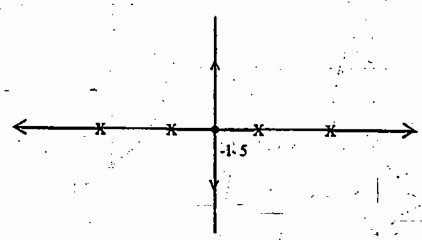
Step 4 : Centroid $= \dfrac{0-1-2-3}{4} = -1.5$

Step 5 : Breakaway point

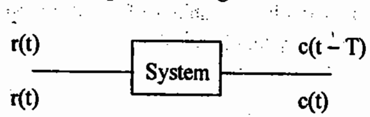
$$K = -s^4 - 6s^3 - 11s^2 - 6s$$

$$\frac{dK}{ds} = 0 \quad \Rightarrow s = -1.5, -0.381, -2.619$$

Here, – 1.5 lies in the root locus and – 0.381, & 2.619 lies in the complementary root locus.

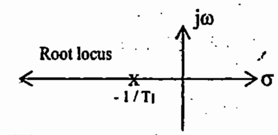
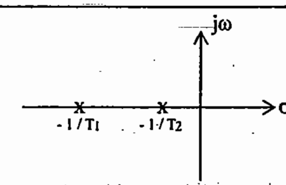
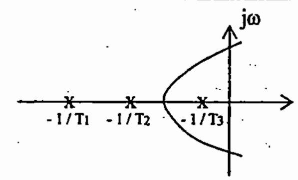
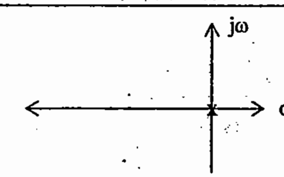
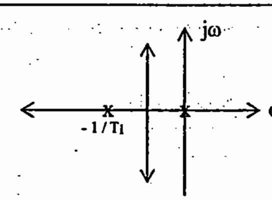
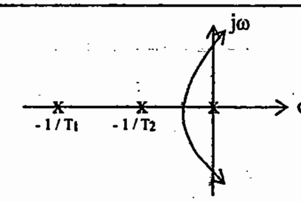


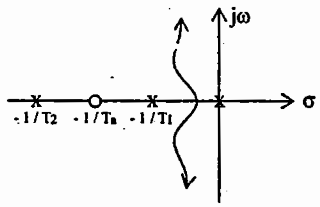
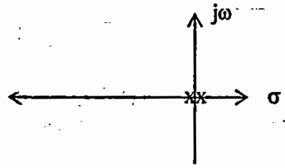
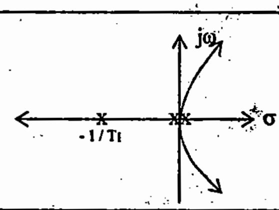
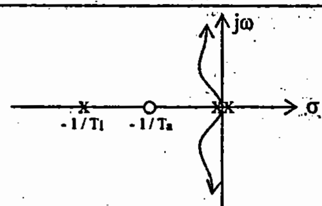
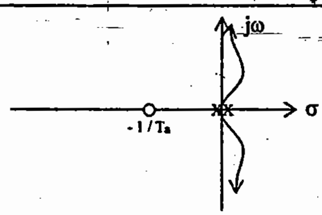
RL of system with transportation lag :



$$\text{Transfer function} = \frac{L[\,\text{Output}\,]}{L[\,\text{Input}\,]} = \frac{C(s)\,e^{-st}}{R(s)}$$

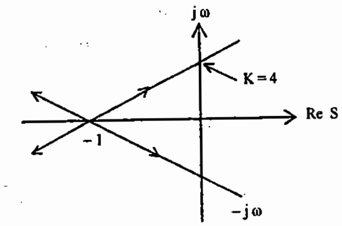
Root Locus Plots for Typical Transfer Functions :

G(s)	Root Locus
1. $\dfrac{K}{sT_1+1}$	
2. $\dfrac{K}{(sT_1+1)(sT_2+1)}$	
3. $\dfrac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$	
4. $\dfrac{K}{s}$	
5. $\dfrac{K}{s(sT_1+1)}$	
6. $\dfrac{K}{s(sT_1+1)(sT_2+1)}$	

7. $\dfrac{K(sT_a+1)}{s(sT_1+1)(sT_2+1)}$	
8. $\dfrac{K}{s^2}$	
9. $\dfrac{K}{s^2(sT_1+1)}$	
10. $\dfrac{K(sT_a+1)}{s^2(sT_1+1)}$	
11. $\dfrac{K(sT_a+1)}{s^3}$	

OBJECTIVE QUESTIONS

1. The root-locus plot is shown alongside. What is the transfer function ?



(a) $\dfrac{4}{s+1}$ (b) $\dfrac{4}{(s+1)^2}$

(c) $\dfrac{4}{(s+1)^3}$ (d) $\dfrac{4}{(s+1)^4}$

02. The asymptotes and the break point coincide at s = – 2. The transfer function can be

(a) $\dfrac{K}{(s+1)(s+2)}$

(b) $\dfrac{K(s+2)}{(s+1)(s+2)}$

(c) $\dfrac{K}{(s+1)(s+2)(s+3)}$

(d) $\dfrac{K}{(s+2)^3}$

03. The transfer function is

$$\frac{K}{(s+1)(s+2)(s+3)}$$

The break point will lie between

a) 0 and – 1 b) – 1 and – 2
c) – 2 and – 3 d) beyond – 3

04. A unity feedback system has an open loop transfer function

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

The centroid of the asymptotes of the root locus plot lies at

a) –4 b) – (4/3)
c) – 13 d) – 10

05. The open-loop transfer function of an unity feedback system is given by

$$G(s) = \frac{5}{(s+1)(s+2)(s+3)}$$

The number of asymptotes of the root locus plot that tend to infinity is given by

a) 3 b) 1 c) 2 d) 4

06. The open-loop transfer function of an unity feedback system is given by

$$G(s) = \frac{5}{s(s+1)(s+2)}$$

The breakaway point of the root locus plot is given by

a) – 0.423 b) – 0.523
c) – 0.700 d) – 0.5

07. A unity feedback system has an open-loop transfer function

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

The angle of asymptotes are given by

a) 45°, 135°, 225° b) 60°, 180°, 300°
c) 90°, 180°, 270° d) 45°, 90°, 135°

08. The open-loop transfer function of a feedback system is

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

The four branches of root-locus originate at

a) – 2, – 3, – 1 + j4, – 1 – j4
b) – 1, – 2, – 3 + j4, – 3 – j4
c) 0, – 4, – 2 + j4, – 2 – j4
d) 0, – 2, – 1 + j4, – 1 – j4

09. The open-loop transfer function of a feedback system is
The real axis segments of the root locus lies between
a) 0 and –1; –2 and –3; – 4 and – ∞
b) –1 and –2; –3 and – 4
c) –1and –1.5; –3.5 and – 4
d) 1 and 2; 3 and 4

10. The main objective of drawing the root locus is
a) To find the time-response of the system
b) To find the frequency response of the system
c) To find the roots of the characteristic equation for different values of system parameters

11. There are three zeros and two poles of GH(s). There will be
a) Three root loci b) Two root loci
c) Five root loci d) One root locus

12. The root loci
a) Start from zeros and end at poles
b) Start from and end at infinity
c) Start from poles and end at zeros and infinity
d) Start from zeros and end at poles and infinity

13. For the root locus, the phase angle criterion is
a) Odd multiple of 180°
b) Even multiple of 180°
c) Odd multiple of 90°
d) None of the above

14. For positive value of K and negative feedback, the root loci exist on the real axis only in those parts
a) Where odd number of poles and zeros present to the right of a point.
b) Where even number of poles and zeros present to the right of the point.
c) Where odd number of poles and zeros present to the left of the point.
d) Where even number of poles and zeros present to the left of the point.

15. For negative feedback system, for complementary root locus (K varied from 0 to –∞), the phase angle criterion is
a) Odd multiple of 180°
b) Even multiple of 180°
c) Odd multiple of 90°
d) 270° only

16. The intersection of the asymptote is given by

(A) $x = \dfrac{\sum \text{Poles of GH(s)} - \sum \text{Zeros of GH(s)}}{p - z}$

(B) $x = \dfrac{\sum \text{Zeros of GH(s)} - \sum \text{Poles of GH(s)}}{p - z}$

C) $x = \dfrac{\text{Poles of GH(s)} + \sum \text{Zeros of GH(s)}}{p - z}$

(D) $x = \dfrac{\sum \text{Zeros of GH(s)} - \sum \text{Poles of GH(s)}}{p + z}$

17. The angles which the asymptotes make with the real axis is given by

(a) $\phi = \dfrac{(2n+1)\pi}{(p+z)}$

(b) $\phi = \dfrac{(2n-1)\pi}{(p+z)}$

(c) $\phi = \dfrac{(2n+1)\pi}{(p-z)}$

(d) $\phi = \dfrac{(2n-1)\pi}{(p-z)}$

18. The intersection of root locus with the imaginary axis is obtained
a) By putting jω = 0 in GH(s)
b) By putting real part of GH(s) = 0
c) From Routh array of 1 + GH(s) find, critical value of K and find ω from the auxiliary equation of a row.
d) From the Routh array get auxiliary equation and get K

19. The break away points are obtained by
a) Putting 1 + GH(jω) = 0 and solving for ω
b) Putting GH(jω) = 0 and solving for ω
c) Differentiating 1 + GH(s) with respect to s and equating (dK/ds) = 0
d) Differentiating 1 + GH(s) with respect to ω and equating (dK/dω) = 0

20. The spirule is used
a) To draw the root locus only
b) To draw the root locus and calibrate it in terms of variable parameter
c) To find the closed loop roots only
d) To find the damping ratio only

Key for Objective Questions :

1. d	2. d	3. b	4. b	5. c	6. a
7. b	8. c	9. a	10. c	11. a	12. c
13. a	14. a	15. b	16. a	17. c	18. c
19. c	20. b				

JTO PREVIOUS QUESTIONS:

01. The root of a system has three asymptotes. The system can have.
(A) four poles and one zero
(B) three poles
(C) five poles and two zeros
(D) all of these

02. Root locus diagram can be used to determine
(A) conditional stability
(B) absolute stability
(C) relative stability
(D) none

03. The value of K for which the system $s^3 + 3s^2 + 3s + 1 + k = 0$ becomes stable is
(A) K = 8 (B) K = 7
(C) K > 7 (D) None

04. The transfer function of a closed – loop system is this system is
(A) marginally stable
(B) conditionally stable
(C) unstable (D) stable

05. The intersection point of the asymptotes for G (s) H (s) = K / s (s + 5) (s + 10) is
(A) 0 (B) -5
(C) -10 (D) -∞

06. The angle of departure at complex conjugate pole -1 + j1 to the system G (s) H (s) = k / $(s^2 + 2s + 2)$ is
(A) +180° (B) -180°
(C) +90° (D) -90°

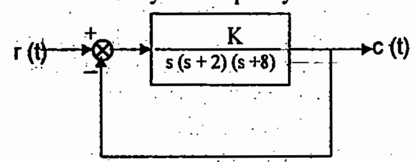
07. A root locus is symmetrical about
(A) imaginary axis
(B) real axis
(C) Both real and imaginary axis
(D) None

Previous PSU's Question

01. The characteristics equation of a system is given by $3s^4 + 10s^3 + 5s^2 + 2 = 0$. This system is
(A) stable (B) marginal stable
(C) unstable
(D) data is insufficient

02. A control system has
$G(S).H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$
for (0<K<∞).
What is the number of breakaway points in the root locus diagram?
(A) One (B) Two
(C) Three (D) Four

03. By a suitable choice of the scalar parameter 'K' the system shown in figure can be made to oscillate continuously at a frequency of



(A) 1 rad/sec (B) 2 rad/sec
(C) 4 rad/sec (D) 8 rad/sec

04. The intersection of asymptotes of root loci of a system with open loop transfer function is

$$G(s).H(s) = \frac{K}{s(s+1)(s+3)}$$

(A) 1.44 (B) 1.33
(C) -1.44 (D) -1.33

05. The value of 'K' for which the unity feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

Crosses the imaginary axis is
(A) 2 (B) 4
(C) 6 (D) 48

06. The root locus of the system

$$G(s).H(s) = \frac{K}{s(s+2)(s+3)}$$

has the break – away point located at
(A) (-0.5,0) (B) (-2.548,0)
(C) (-4,0) (D) (-0.784,0)

07. The transfer function of the system is

$$\frac{2s^2+6s+5}{(s+1)^2(s+2)}$$

, the characteristic equation of the system is
(A) $2s^2+6s+5=0$
(B) $(s+1)^2(s+2)=0$
(C) $2s^2+6s+5+(s+1)^2(s+2)=0$
(D) $2s^2+6s+5(s+1)^2(s+2)=0$

08. Which one of the following techniques is utilized to determine the actual point at which the root locus crosses the imaginary axis?
(A) Nyquist technique
(B) Routh – Hurwitz criterion
(C) Nichol's criterion
(D) Bode techniquie

09. The root locus plot of the system having the loop transfer function

$$G(s).H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$$

has
(A) no breakaway point
(B) three real breakaway point
(C) only one breakaway point
(D) one real and two complex breakaway points

10. The open – loop transfer function of unity feedback control system is

$$G(s) = \frac{K}{s(s+a)(s+b)}, \quad 0 < a \le b.$$

The system is stable, if

(A) $0 < K < \frac{(a+b)}{ab}$
(B) $0 < K < \frac{ab}{(a+b)}$
(C) $0 < K < ab(a+b)$
(D) $0 < K < \frac{a}{b}(a+b)$

KEYS :

JTO:

(1) D (2) C (3) D (4) C (5) B

(6) C (7) B

PSU'S :

(1) C (2) C (3) C (4) D (5) D

(6) D (7) B (8) B (9) B (10) C

CHAPTER – 5 FREQUENCY RESPONSE ANALYSIS

The various frequency response analysis techniques are
1) Bode plot
2) Polar plot
3) Nyquist plot
4) M & N circles
5) Nicholas chart

1) Bode plots :

It is used to draw the frequency response of a open loop and closed-loop system. The representation of the logarithm of $|G(j\omega)|$ and phase angle of $G(j\omega)$, both plotted against frequency in logarithmic scale. These plots are called Bode plots.

Bode Plot of first order system :

Let the Transfer Function $= \dfrac{1}{1+Ts}$

subs. $s = j\omega$

$$T.F. = \frac{1}{1+j\omega T}$$

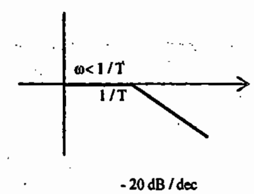
$$M = \frac{1}{\sqrt{1+(\omega T)^2}} \; ; \qquad \phi = -\tan^{-1}(\omega T)$$

$$M = 20\log\frac{1}{\sqrt{1+(\omega T)^2}} = -10\log[1+(\omega T)^2]$$

ω

$\omega << 1/T$: $M_{dB} \cong 10\log 1 \cong 0$

$\omega << 1/T$: $M_{dB} = -10\log(\omega T)^2 = -20\log\omega T$



Therefore, the error at the corner frequency $\omega = 1/T$ is

$$-10\log 2 + 10\log 1 = -3\text{ dB}$$

The error at frequency ($\omega = 1/2T$) one octave below the corner frequency is

$$-10\log(1+¼) + 10\log 1 = -1\text{ Db}$$

Bode Plot of second order system :

$$\text{T.F.} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

subst. $s = j\omega$

$$\text{T.F.} = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2}$$

Divide with 'ω_n^2'

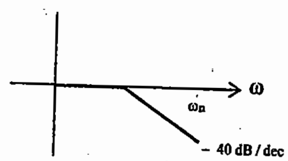
$$= \frac{1}{(1-\mu^2)^2 + j2\zeta\mu} \qquad [\mu = \omega/\omega_n]$$

$$\therefore \quad M = \frac{1}{\sqrt{(1-\mu^2)^2 + (2\zeta\mu)^2}} \; ; \quad \phi = -\tan^{-1}\left(\frac{2\zeta\mu}{1-\mu^2}\right)$$

$$M_{dB} = -10\log\left[(1-\mu^2)^2 + 4\zeta^2\mu^2\right]$$

Case 1) When $\mu < 1 \Rightarrow (\omega/\omega_n) < 1 \Rightarrow \omega < \omega_n$

$$M_{dB} \cong -10\log 1 \cong 0\text{ dB}$$



Case 2) When $\mu > 1 \Rightarrow (\omega/\omega_n) > 1 \Rightarrow \omega > \omega_n$

$$M_{dB} \cong -10\log\mu^4 \cong -40\log\mu$$

The error between the actual magnitude and the asymptotic approximation is as given below.

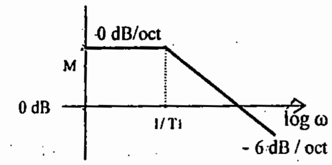
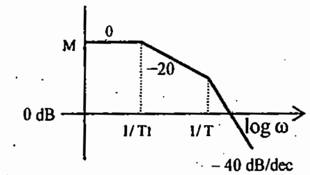
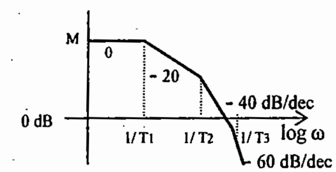
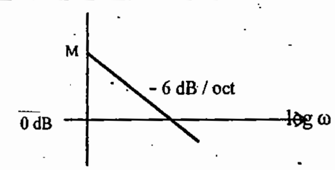
For $0 < \mu << 1$, the error is

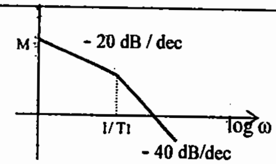
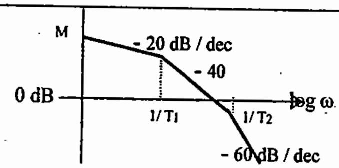
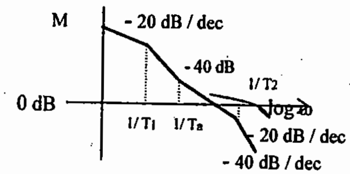
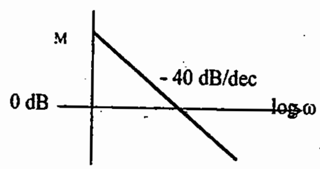
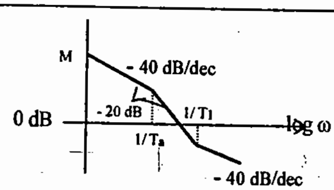
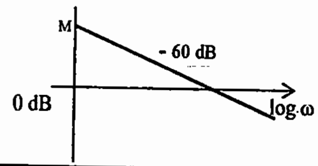
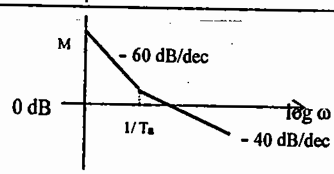
$$-10\log[(1-\mu^2)^2 + 4\zeta^2\mu^2] + 10\log 1$$

and for $1 < \mu << \infty$, the error is

$$-10\log[(1-\mu^2)^2 + 4\zeta^2\mu^2] + 40\log\mu$$

Bode Plots for Typical Transfer Functions :

G(s)	Bode Plot
1. $\frac{K}{sT_1 + 1}$	
2. $\frac{K}{(sT_1+1)(sT_2+1)}$	
3. $\frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$	
4. $\frac{K}{s}$	

5. $\dfrac{K}{s(sT_1+1)}$	
6. $\dfrac{K}{s(sT_1+1)(sT_2+1)}$	
7. $\dfrac{K(sT_a+1)}{s(sT_1+1)(sT_2+1)}$	
8. $\dfrac{K}{s^2}$	
9. $\dfrac{K(sT_a+1)}{s^2(sT_1+1)}$	
10. $\dfrac{K}{s^3}$	
11. $\dfrac{K(sT_a+1)}{s^3}$	

2) Polar plot :

The sinusoidal transfer function $G(j\omega)$ is a complex function and is given by

$$G(j\omega) = \text{Re}\,[G(j\omega)] + j\,I_m\,[G(j\omega)]$$

or $$G(j\omega) = |G(j\omega)|\,\angle G(j\omega) = M\angle\phi$$

from above equation, it is seen that $G(j\omega)$ may be represented as a phasor of magnitude M and phase angle ϕ. As the input frequency ω is varied from 0 to ∞, the magnitude M and phase angle ϕ change and hence the tip of the phasor $G(j\omega)$ traces a locus in the complex plane. The locus thus obtained is known as *polar plot*.

When a transfer function consists of 'P' poles and 'Z' zeros, and it doesn't consist poles at origin then the polar plot starts from 0^0 with some magnitude and terminates at $-90^0 \times (P - Z)$ with zero magnitude.

When a transfer function consists of poles at origin, then the polar plot starts from $-90^0 \times$ no. of poles at origin with '∞' magnitude and ends at $-90^0 \times (P - Z)$ with zero magnitude.

2) Nyquist Stability Criteria :

It is used to determine the stability of a closed-loop system using polar plots. This concept is derived from complex analysis using 'Principle of Argument'.

Let $$G(s) = \frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)} \longrightarrow (1)$$

Characteristic Equation, i.e, $1 + G(s)$

$$1 + G(s) = 1 + \frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$$

$$= \frac{(s+P_1)(s+P_2) + (s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)} \longrightarrow (2)$$

From (1) and (2), the open-loop poles and CE poles are same.

$$\text{C.E.} = \frac{(s + Z_1^1)(s + Z_2^1)}{(s + P_1)(s + P_2)} \longrightarrow (3)$$

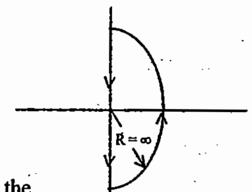
$$\text{Overall transfer function} = \frac{G(s)}{1 + G(s)} = \frac{(s + Z_1)(s + Z_2)}{(s + Z_1^1)(s + Z_2^1)} \longrightarrow (4)$$

From (3) and (4), the C.E zeros and closed-loop poles are same.

→For the closed-loop system to be stable, the zeros of the C.E should not be located on the right half of the s-plane.

Using Principle of Argument

$$Q(s) = 1 + G(s)$$



Consider a contour as shown which covers the entire right half of the s-plane. If each and every point is along the boundary of the contour is substituted in C.E according to the principle of argument.

The no. of encircles with respect to origin, $N = Z - P$

where Z and P are the zeros and poles of the C.E located inside the contour or located in right half of the s-plane.

For the closed-loop system to be stable, $Z = 0$.

→For the open-loop system to be stable, $P = 0$, then $N = Z$.

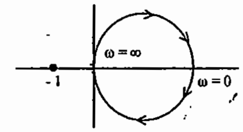
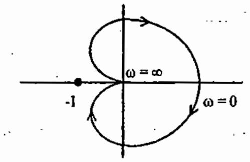
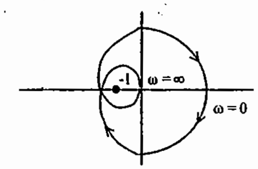
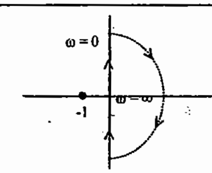
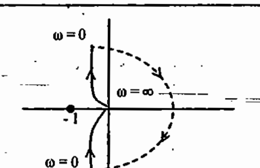
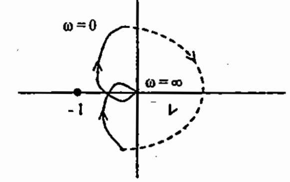
In $N = Z - P$, Z becomes '0' only if $N = 0$ [q(s) contour shouldn't encircle the origin]

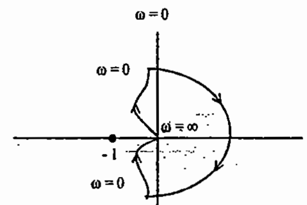
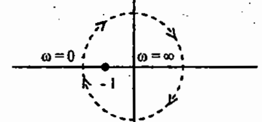
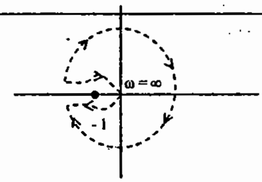
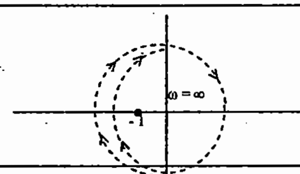
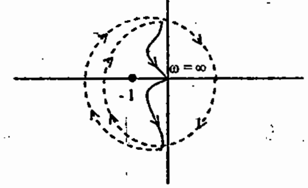
If the open-loop system is stable, the closed-loop system will be stable only if the Nyquist contour doesn't encircles origin.

→For the open-loop system to be unstable, $P \neq 0$.

If the open-loop system is unstable, the closed-loop system will be stable only if the Nyquist contour encircles origin in clockwise direction. The number of encirclements should be equal to the number of open-loop poles located inside the contour.

Nyquist Plots for Typical Transfer Functions :

G(s)	Nyquist Plot
1. $\dfrac{K}{s T_1 + 1}$	
2. $\dfrac{K}{(s T_1 + 1)(s T_2 + 1)}$	
3. $\dfrac{K}{(s T_1 + 1)(s T_2 + 1)(s T_3 + 1)}$	
4. $\dfrac{K}{s}$	
5. $\dfrac{K}{s (sT_1 + 1)}$	
6. $\dfrac{K}{s (s T_1 + 1)(s T_2 + 1)}$	

7. $\dfrac{K(sT_a+1)}{s(sT_1+1)(sT_2+1)}$	
8. $\dfrac{K}{s^2}$	
9. $\dfrac{K(sT_a+1)}{s^2(sT_1+1)}$	
10. $\dfrac{K}{s^3}$	
11. $\dfrac{K(sT_a+1)}{s^3}$	

4) M & N Circles

Constant Magnitude Loci : M-Circles

M – circles are used to determine the magnitude response of a closed-loop system using open-loop transfer function.

It is applicable only for unity feedback system. The open-loop transfer function G (jω) of a unity feedback control system is a complex quantity and can be expressed as

$$G(j\omega) \, . \, 1 = x + jy$$

Since
$$M = \frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)}$$

$$\therefore \quad M = \frac{x+jy}{1+x+jy}$$

$$\therefore \quad M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

On squaring on both sides and simplifying following equation is obtained :

$$(1-M^2)x^2 - 2M^2x + (1-M^2)y^2 = M^2$$

or
$$x^2 - \frac{2M^2}{(1-M^2)}x + y^2 = \frac{M^2}{(1-M^2)}$$

Add $\left(\dfrac{M^2}{1-M^2}\right)^2$ to both sides ,

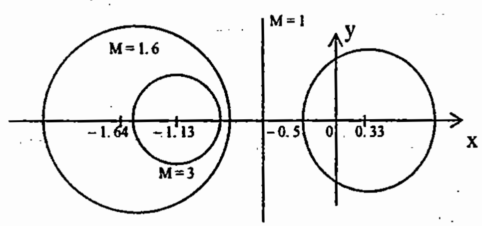
$$\therefore \quad x^2 - \frac{2M^2}{(1-M^2)}x + \left(-\frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2}{(1-M^2)} + \left(\frac{M^2}{1-M^2}\right)^2$$

or
$$\left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2$$

For different values of M, above Eq. represents a family of circles with centre at $x = (M^2/1-M^2)$, $y = 0$ and radius as $(M^2/1-M^2)$. On a particular circle the value of M (magnitude of closed-loop transfer function) is constant, therefore, these circles are called M-circles.

The centres and radii of M-circles for different values of M are given in the following table and M-circles are drawn in the following figure.

M	centre $x = M^2/1 - M^2, y = 0$	Radius $r = M/1 - M^2$
0.5	0.33	0.67
1.0	∞	∞
1.2	−3.27	2.73
1.6	−1.64	1.03
2.0	−1.33	0.67



In G (jω) plane the Nyquist plot is superimposed on M-circle and the points of intersection that gives the magnitude of C (jω) / R (jω) at different values of 'ω'.

Constant Phase Angles Loci : N-circles:

N – circles are used to determine the phase response of a closed-loop system using open-loop transfer function.

The phase angle of the closed-loop transfer function of a unity feedback system is given by

$$\angle \frac{C(j\omega)}{R(j\omega)} = \angle \frac{x + jy}{1 + x + xy}$$

The phase angle is denoted by ϕ, therefore,

$$\phi = \tan^{-1}(y/x) - \tan^{-1}[y/(1+x)]$$

or $$\tan\phi = \tan\left\{\tan^{-1}(y/x) - \tan^{-1}[y/(1+x)]\right\}$$

$$= \frac{\tan\left(\tan^{-1}(y/x)\right) - \tan\left(\tan^{-1}[y/(1+x)]\right)}{1 + \tan\left(\tan^{-1}(y/x)\right).\tan\left(\tan^{-1}[y/(1+x)]\right)}$$

$$= \frac{(y/x) - [y/(1+x)]}{1 + (y/x).[y/(1+x)]}$$

or $$\tan\phi = \frac{y}{x^2 + x + y^2}$$

Substituting $\tan\phi = N$ in above equation

$$\therefore \quad N = \frac{y}{x^2 + x + y^2}$$

or $$x^2 + x + y^2 - (y/N) = 0$$

Add $\left(\frac{1}{4} + \frac{1}{4N^2}\right)$ on both sides

$$\therefore \left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - \frac{y}{N} + \frac{1}{4N^2}\right) = \left(\frac{1}{4} + \frac{1}{4N^2}\right)$$

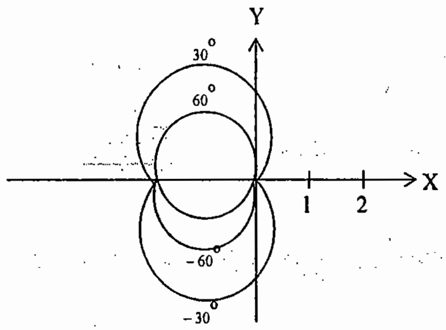
or $$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\frac{1}{4} + \frac{1}{4N^2}\right)$$

For different values of N, above equation represents a family of circles with centre at x = - ½ , y = 1 / 2 N and radius as

$$\sqrt{\frac{1}{4} + \frac{1}{4N^2}}$$

On a particular circle the value of N or the value phase angle of the closed-loop transfer function is constant , therefore, these circles are called N-circles.

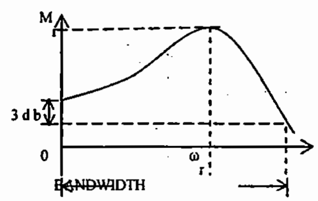
ϕ	N = tan ϕ	center x = - ½ ,y = 1 / 2N	Radius $R = \sqrt{1/4 + 1/4N^2}$
−90°	∞	0	0.5
−60°	−1.732	−0.289	0.577
−50°	−1.19	−0.42	0.656
−30°	−0.577	0.866	1.0
−10°	−0.176	−2.84	2.88
0°	0	∞	∞
+10°	0.176	2.84	2.88
+30°	0.577	0.866	1.0
+50°	1.19	0.42	0.656
+60°	1.732	0.289	0.577
+90°	∞	0	0.5



Cutoff frequency and Bandwidth :

The closed-loop frequency response of a system is shown in the figure. The response falls by 3 dB from its low frequency value to a frequency value ω_c. The frequency ω_c is called cut
Off frequency and the frequency range 0 to ω_c is called the bandwidth of the system. The resonant
Peak M_r occurs at resonance frequency ω_r.

The bandwidth is defined as the frequency at which The magnitude gain of the frequency response plot reduces to $1/\sqrt{2} = 0.707$; i.e. 3 db of its low frequency value.



For a second order system

$$M(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The bandwidth of a second order system having non-zero magnitude at $\omega = 0$ is given by

$$\text{B.W.} = \omega_n \left(1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}\right)^{1/2}$$

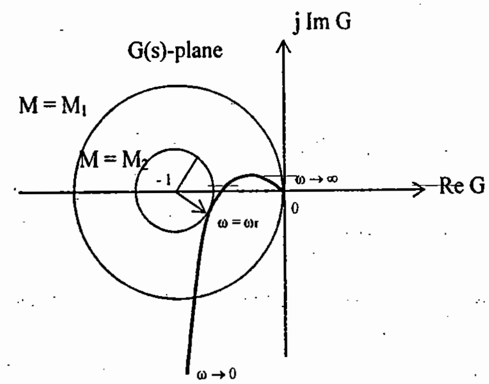
The resonant frequency is $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

The resonant magnitude is

$$M_r = \frac{1}{2\zeta\sqrt{(1-\zeta^2)}}$$

5) Nichols Chart

The transformation of constant – M and constant – N circles to log-magnitude and phase angle coordinates and the resulting chart is known as the Nichols chart.

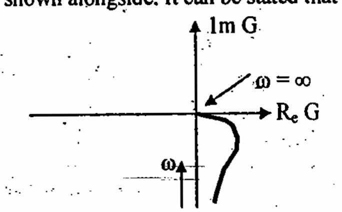


OBJECTIVE QUESTIONS

01. In a polar plot, the curve was found to cross the negative real axis at –1.2. Then
   a) The gain margin is 1.2 and the system is stable.
   b) The gain margin is 0.833 and the system is stable.
   c) The gain margin is 1.2 and the system is unstable.
   c) The gain margin is 0.83 and the system is unstable.

02. The polar plot of a closed-loop system with a transfer function $\frac{G}{1+GH}$ is drawn
   a) G b) 1 + GH
   c) GH d) $\frac{G}{1+GH}$

03. The polar plot of a transfer function is shown alongside. It can be stated that



   a) the finite zero is closer to the origin than the finite pole
   b) the finite pole is closer to the origin than the zero is
   c) the system is unstable
   d) the system is non-minimum phase

04 The polar plot is drawn
   a) decibel versus ω
   b) decibel versus log ω
   c) decibel versus phase angle
   d) magnitude and phase incorporated in the x – y plane.

05. For a transfer function with pure transportation lag, the polar plot is

a) a semi circle with centre at (–1, 0) and radius 1 in the clockwise direction.
b) a semi circle with centre at (– 1, 0) and radius 1 in the anti-clockwise direction.
c) a circle with origin as the centre and radius 1.
d) polar plot does not exist.

06. By substituting $s = j\omega$, the frequency response plot gives

a) transient response of the system
b) steady state response of the system
c) initially transient and then steady state response
d) none of the above

07. The polar plot of a system with transfer function $G(s) = \frac{K}{s(s+T)}$

for +ve T and –ve K will be

a) in the first quadrant
b) in the second quadrant
c) in the third quadrant
d) in the fourth quadrant

08. Which of the following transfer function is a non-minimum phase transfer function ?

(a) $\frac{(s+1)}{(s+2)(s+3)(s+4)}$

(b) $\frac{(s+1)(s+2)}{s(s+3)(s+4)}$

(c) $\frac{10(s+1)}{s^2(s+3)}$

(c) $\frac{(s+1)}{(s+2)(s+3)}$

09. In the $G^{-1}(j\omega)$ plane, the constant phase angle loci are

a) straight lines passing through the origin
b) straight lines passing through (–1, 0) point
c) straight lines passing through (1, 0) point
d) some pass through (–1, 0), some pass through (1, 0) and some pass through origin.

10. The inverse polar plot is the plot of the following sinusoidal transfer function

a) $G(1/j\omega)$

b) $\frac{1}{G(j\omega)}$

c) $\frac{1}{G(1/j\omega)}$

d) None

Key :

1. d 2. c 3. a 4. d 5. c 6. b
7. a 8. d 9. a 10. b

OBJECTIVE QUESTIONS

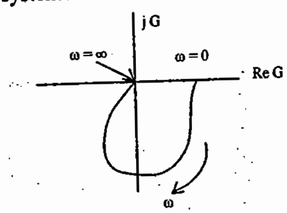
01. A stable feedback control system has open-loop transfer function with a pole at RHP and zero at RHP. The corresponding Nyquist plot will

a) encircle (–1, j0) point in counter clockwise direction once.
b) have anti-clockwise encirclement of the (–1, j0) point once.
c) encircle (–1, j0) point as many times in the counter clockwise direction as the number of LHP poles of the closed-loop transfer function.
d) no encircle (–1, j0) point at all.

02. Nyquist plot can be used

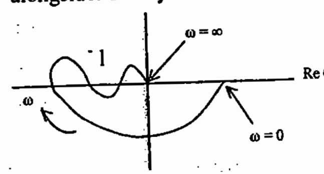
a) only to find the closed-loop poles in the right half plane.
b) ascertain the stability only.
c) to find the open-loop poles in the right half plane.
d) to find the number of closed-loop poles in the left half plane.

03. Nyquist plot of a system is shown alongside. What is the type of the system?



a) 0 b) 1 c) 2 d) 3

04. Nyquist plot of a system is shown in alongside. The system is



a) marginally stable
b) conditionally stable
c) stable
d) unstable

05. The unit circle of the Nyquist plot transforms into 0 db line of the amplitude plot of the Bode diagram is

a) at zero frequency
b) at low frequency
c) at high frequency
d) at any frequency

06. A unit feedback system has an open-loop transfer function

$$G(j\omega) = \frac{K}{j\omega(j0.2\omega+1)(j0.05\omega+1)}$$

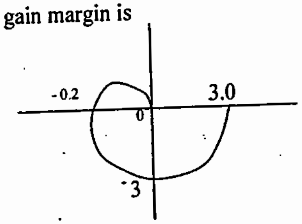
The phase cross-over frequency of the Nyquist plot is given by

a) 1 rad / sec b) 100 rad / sec
c) 10 rad / sec d) 0.1 rad / sec

07. If the Nyquist plot of a certain feedback system crosses the negative real axis of – 0.1 point, the gain margin of the system is given by

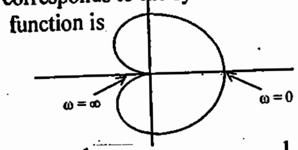
a) 0.1 b) 10
c) 100 d) 200

08. The Nyquist plot (for positive frequencies) of the open-loop transfer function is shown in figure. The gain margin is



a) 2 b) 3 c) 4 d) 5

09. The Nyquist plot given figure corresponds to the system whose transfer function is



(a) $\frac{1}{(s+1)^3}$ (b) $\frac{1}{(s+1)^2}$

(c) $\frac{1}{s^2+2s+2}$ (d) $\frac{1}{s+1}$

10. If the gain margin of a certain feedback system is given as 20 dB, the Nyquist plot will cross the negative real axis at the point
 a) s = − 0.05 b) s = − 0.2
 c) s = − 0.1 d) s = − 0.01

Key :
1. d 2. d 3. a 4. b 5. d 6. c
7. b 8. d 9. b or c 10. c

Objective Questions

01. If the phase ϕ which is the angle between radial line connecting a pole and origin is equal to 45°, then the peak overshoot is
 a) 0.5 % b) 1.2 %
 c) 3 % d) 4.32 %

02. The denormalised bandwidth for a particular value of ω_n and damping factor ζ is
 a) $\omega_n\sqrt{1-\zeta^2}$ b) $(\omega_n+\zeta^2)^{3/2}$
 c) $\omega_n\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]$
 d) $\omega_n \; 1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4} \;\; ^{1/2}$

03. The resonant and damping frequency of a certain system was found to be 7.07 rad/s and 8.666 rad /s respectively. The real co-ordinate of the dominant pole is :
 a) − 8.12 b) − 7
 c) − 6.65 d) − 5

04. Large bandwidth corresponds to
 a) small rise time and suppresses noise
 b) small rise time and increases noise
 c) high rise time and suppresses noise
 d) high rise time and increases noise

05. The resonant peak of a second order system is given by
 a) $M_P = \exp(-\zeta\omega_n/\sqrt{1-2\zeta^2})$
 b) $M_P = \omega_n\sqrt{1-\zeta^2}$
 c) $M_P = \dfrac{1}{\zeta\sqrt{1-2\zeta^2}}$
 d) $M_P = \dfrac{1}{2\zeta\sqrt{1-2\zeta^2}}$

06. The centre and radius of a constant M circles are given by
 a) $x = \dfrac{M}{(1-M^2)}$, $y = 0$; $r = \dfrac{M}{(1-M^2)}$
 b) $x = \dfrac{M}{(M^2-1)}$, $y = 0$; $r = \dfrac{M}{(M^2-1)}$
 c) $x = \dfrac{M^2}{(M^2-1)}$, $y = 0$; $r = \dfrac{M}{(M^2-1)}$
 d) $x = \dfrac{M^2}{(M^2-1)}$, $y = 0$; $r = \dfrac{M}{(M^2-1)}$
 M M

07. The centre and radius of a constant N circles are given by
 a) $x = -(1/2)$, $y = (1/2N)$; $r = \dfrac{\sqrt{(N^2+1)}}{2N}$
 b) $x = -(1/2)$, $y = (1/2N)$; $r = \dfrac{\sqrt{(N^2+1)}}{2N}$
 c) $x = -(1/2)$, $y = -(1/2N)$; $r = \dfrac{\sqrt{(N^2+1)}}{2N}$
 d) $x = -(1/2)$, $y = -(1/2N)$; $r = \dfrac{\sqrt{(N^2+1)}}{2N}$

08. In the $G^{-1}(j\omega)$ plane, the constant M circles has the following centre and radius.
 a) (x, y) = (−1, 0) ; r = M
 b) (x, y) = (+1, 0) ; r = M
 c) (x, y) = (−1, 0) ; r = 1/ M
 d) (x, y) = (+1, 0) ; r = 1/ M

09. Undamped natural frequency ω_n and resonance frequency ω_r of a unity feedback system with open-lop transfer function (M. Gopal)
 $$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \; ; \; \zeta < 1/\sqrt{2},$$
 are related as
 a) $\omega_n = \omega_r$ b) $\omega_n > \omega_r$
 c) $\omega_n < \omega_r$ d) None

10. Undamped natural frequency ω_n and bandwidth ω_b of a unity feedback system with open-lop transfer function
 $$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$
 are related as
 a) $\omega_n = \omega_b$ b) $\omega_n > \omega_b$
 c) $\omega_n < \omega_b$ d) None

Key :
1. d 2. d 3. d 4. b 5. d 6. d
7. a 8. c 9. b 10. c

JTO PREVIOUS QUESTION:

01. The bode plot is applicable to network
 (A) all phase
 (B) maximum phase
 (C) minimum phase
 (D) none

02. Nyquist criterion is used to find which of the following
 (A) Relative stability
 (B) Absolute stability
 (C) Both A and B
 (D) None

03. The frequency range specification required to satisfactorily describe the system responsible is?
 (A) resonance (B) band width
 (C) voltage (D) all of the above

04. Band width of a control system is used as a means of specifying its performance relating to ?
 (A) stability of the system
 (B) speed of the system
 (C) constant gain of the system
 (D) none

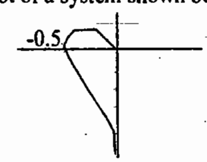
05. ________ can be extended to systems which are time – varying?
 (A) Root locus design
 (B) Bode – Nuquist stability methods
 (C) State model representatives
 (D) Transfer functions

06. When gain K of the loop transfer function is varied from zero to infinity the closed loop system
 (A) stability is improved
 (B) always become unstable
 (C) Stability is not affected
 (D) may become unstable

07. Frequency domain analysis is preferred when dealing with systems having input is
 (A) ramp and parabolic
 (B) sinusoidal with fixed frequency
 (C) sinusoidal with variable frequency and amplitude
 (D) non – sinusoidal with lagging power factor

08. For gain K log – magnitude curve in Bode polt is
 (A) negative or positive depending upon the value of K
 (B) Zero
 (C) negative
 (D) positive

09. What is the type of the system, for the Nyquist plot of a system shown below?
 (A) 1
 (B) 0
 (C) 2
 (D) 3



10. Which of the following statements is not true for Nyquist criterian?

(A) It indicates the degree of stability of a stable system
(B) It's provides some amount of information about absolute stability as the Routh Criterion
(C) Either A or B
(D) None

11. A minimum phase function has m finite poles and n finite Zeros. Its phase angle at infinity is

(A) ∏ radians
(B) (n – m) ∏/2 radians
(C) (m – n) ∏/2 radians
(D) none

12. A transfer function which has one or more zeros in the R.H.S. plane is known as____ transfer function

(A) all phase
(B) minimum phase
(C) non – minimum phase
(D) none of the above

PREVIOUS PSU'S QUESTION

01. If the Nyquist plot cuts the negative real axis at a distance of 0.4. The gain margin of the system is

(A) 0.4 (B) -0.4
(C) 4% (D) 2.5

02. A minimum phase unity feedback system has a Bode plot with a constant slope of 0 db/decade for all frequencies. What is the value of the maximum phase margin for the system?

(A) 0^0 (B) 90^0
(C) -90^0 (D) 180^0

03. A minimum phase unity feedback system has a bode plot with a constant slope of -20 db/decade for all frequencies. What is the value of the maximum phase margin for the system?

(A) 0^0 (B) 90^0
(C) -90^0 (D) 180^0

04. The bode – plot is valid for

(A) minimum phase network
(B) all phase network
(C) non – minimum phase network
(D) none of the above

05. The initial slope of the bode – plot gives an indication of

(A) type of the system
(B) nature of the system time response
(C) system stability
(D) gain margin

06. If the magnitude of the polar plot at phase crossover is 'a', the gain margin is

(A) – a (B) 0 (C) a (D) 1/a

07. In the bode – plot of a unity feed back control system, the value of phase of G (jω) at the gain cross over frequency is -125^0. The phase margin of the system is

(A) -125^0 (B) -55^0
(C) $+55^0$ (D) $+125^0$

08. Nichol's chart is useful for detailed study and analysis of

(A) closed loop frequency response
(B) open loop frequency response
(C) close loop and open loop frequency response
(D) none of the above

09. Constant M circles have their centre and radius as

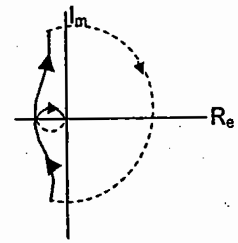
(A) $\left(\frac{-M^2}{M^2-1}, 0\right)$ and $\left(\frac{M^2}{M^2-1}\right)$

(B) $\left(\frac{-M^2}{M^2-1}, 0\right)$ and $\left(\frac{M}{M^2-1}\right)$

(C) $\left(0, \frac{M^2}{M^2-1}\right)$ and $\left(\frac{M^2}{M^2-1}\right)$

(D) $\left(0, \frac{M^2}{M^2-1}\right)$ and $\left(\frac{M}{M^2-1}\right)$

10. Nyquist plot shown in the given figure is for a type



(A) zero system
(B) one system
(C) two system
(D) three system

11. The open loop transfer function of a unity feedback control system is given as

$$G(s) = \frac{1}{(1 + sT_1)(1 + sT_2)}$$

The phase crossover frequency and the gain margin are respectively

(A) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1 + T_2}{T_1 T_2}$

(B) $\sqrt{T_1 T_2}$ and $\frac{T_1 + T_2}{T_1 T_2}$

(C) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1 T_2}{T_1 + T_2}$

(D) $\sqrt{T_1 T_2}$ and $\frac{T_1 T_2}{T_1 + T_2}$

12. The polar plot of a transfer function passes through the critical point (-1,0). Gain margin is

(A) zero (B) -1 dB
(C) 1 dB (D) infinity

KEYS:

JTO

(1) C (2) C (3) B (4) B (5) A

(6) D (7) C (8) A (9) B (10) D

(11) C (12) C

PSU's

(1) D (2) B (3) B (4) A (5) A

(7) C (8) A (9) B (10) B (11) B

(12) B

GAIN MARGIN AND PHASE MARGIN

Gain cross - over frequency: The frequency at which the magnitude equal to one or 0 dB
Phase cross over frequency: The frequency at which the phase angle is equal to -180^0.

Gain margin $GM = \dfrac{1}{|G(j\omega) H(j\omega)|}$ **(In Linear)**

The gain margin is a factor by which the gain of a stable system is allowed to increase before the system reaches instability.

The gain margin in dB is

$$G.M = 20 \log_{10} \frac{1}{|G(j\omega_c) H(j\omega_c)|} \text{ dB}$$

Procedure to calculate Gain margin :

1. Calculate Phase crossover frequency
   a) by equating phase equation to 180^0 or
   b) by equating imaginary part to zero
2. Calculate the maginitude at phase crossover frequency and is equal to 'a'.
3. Gain margin is equal to 20 log (1 / a).
   For stable systems as $|G(j\omega_c) H(j\omega_c)| < 1$, the gain margin in dB is positive.
   For marginally stable systems as $|G(j\omega_c) H(j\omega_c)| = 1$, the gain margin in dB is zero.

   For unstable systems as $|G(j\omega_c) H(j\omega_c)| > 1$, the gain margin in dB is negative an the gain is to be reduced to make the system is stable.

Phase margin :
The phase margin of a stable system is the amount of additional phase lag required to bring the system to the point of instability.
The phase margin is given by $P.M. = 180^0 + \angle G(s) H(s)$

Procedure for calculation of P.M :

1. Calculate 'ω_G' by equating magnitude equation to '1'.
2. Calculate the phase at $\omega = \omega_G$
3. $P.M. = 180^0 + \angle G(s) H(s)$.
4. P.M is positive, the system is stable.
   P.M is negative, the system is unstable.
   P.M is zero, the system is marginally stable.

JTO PREVIOUS QUESTION

01.Phase margin is the amount of angle to make the system.
(A) Oscillatory (B) unstable
(C) exponential (D) stable

02. A system with gain margin close to unity or a phase margin close to zero is
(A) relatively stable
(B) highly stable
(C) oscillatory
(D) none

03. The phase shift of the second order system with transfer function 1/s is
(A) -90 (B) 90
(B) 180 (D) -180

04. A minimum phase unity feedback system has a Bide plot with a constant slope of 0 db/decade for all frequencies. What is the value of the maximum phase margin for the system?
(A) 0^0 (B) 90^0
(C) -90^0 (D) 180^0

05. The gain margin of a unity negative feed back system having forward transfer function is $\dfrac{K}{s(sT+1)}$
(A) infinity (B) KT
(C) 1 (D) Zero

06. If the magnitude of the polar plot at phase crossover is 'a', the gain margin is
(A) – a (B) 0
(C) a (D) 1/a

07. For the transfer function
$$G(s).H(s) = \frac{1}{s(s+1)(s+0.5)}$$
The phase crossover frequency is
(A) 0.5 rad / sec
(B) 0.707 rad / sec
(C) 1.732 rad / sec
(A) 2 rad / sec

08. The open loop transfer function of a feedback control system is
$$G(s).H(s) = \frac{1}{(s+1)^3}$$
The gain margin of the system is
(A) 2 (B) 4
(C) 8 (D) 16

09. The gain margin (in dB) of a system having the loop transfer function
$$G(s).H(s) = \frac{\sqrt{2}}{s(s+1)}$$
(A) 0 (B) 3
(C) 6 (D) α

10. If the gain of the loop system is doubled, the gain margin of the system is
(A) not affected
(B) doubled
(C) halved
(D) one fourth of original value

11. The forward path transfer function of an unity feedback system is given by
$$G(s) = \frac{1}{s(s+1)^2}$$
What is the phase margin ror this system?
(A) $-\pi$ rad (B) 0 rad
(C) $\pi/2$ rad (D) π rad

12. The open loop transfer function of a system is

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$

The phase crossover frequency ω_C is

(A) $\sqrt{2}$ (B) 1
(C) zero (D) $\sqrt{3}$

13. The open loop transfer function of a unity feedback control system is given as The phase crossover frequency and the gain margin are respectively

(A) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1+T_2}{T_1T_2}$

(B) $\sqrt{T_1 T_2}$ and $\frac{T_1+T_2}{T_1T_2}$

(C) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1T_2}{T_1+T_2}$

(D) $\sqrt{T_1 T_2}$ and $\frac{T_1T_2}{T_1+T_2}$

14. The polar plot of a transfer function passes through the critical point (-1,0). Gain margin is

(A) zero (B) -1 dB
(C) 1 dB (D) infinity

KEYS: (1) B (2) C (3) A (4) D (5) D (6) B (7) A (8) B (9) C (10) B (11) C (12) B (13) A (14) B

NOTE:

Minimum phase transfer function : Transfer function have no poles and zeros in the RHS of s-plane.

Non-minimum phase transfer function : Transfer function having at least one pole or zero in the RHS of s-plane.

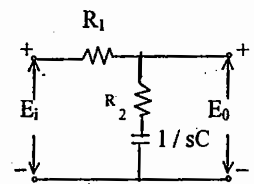
ll pass transfer function : Transfer function have symmetric pole and zero about the imaginary axis in s-plane.

6.1 COMPENSATORS

Lag compensator :

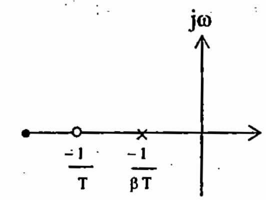
A compensator having the characteristic of a lag network is called a lag compensator. Lag compensation results in a *large improvement in steady state performance* but results in a slower response due to reduced bandwidth. Lag compensator is essentially *a low pass filter* and so high frequency noise signals are attenuated.

Transfer function of lag compensator, $G_c(s) = \frac{s+z_c}{s+p_c}$



Electrical lag compensator

$$\frac{s+(1/T)}{s+(1/\beta T)}$$



Pole-zero plot of lag Compensator.

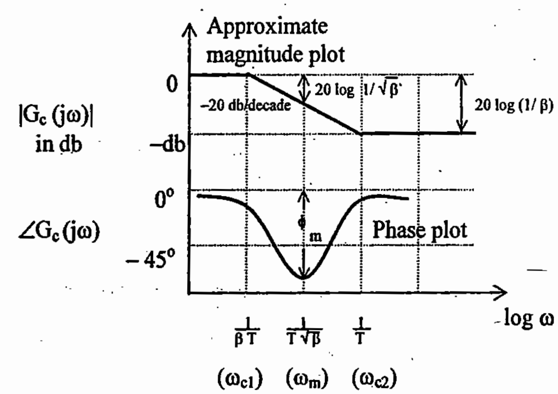


Fig : Bode plot of lag compensator

As 'ω' is varied from 0 to ∞, the phase angle decreases from 0 to a maximum value of ϕ_m At $\omega = \omega_m$, then increases from maximum value to 0.

Frequency of maximum phase lag, $\omega_m = \sqrt{\omega_{c1}\,\omega_{c2}} = \sqrt{(1/\beta T).(1/T)}$

$$= \frac{1}{T\sqrt{\beta}}$$

$$\therefore \text{Maximum lag angle, } \phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)$$

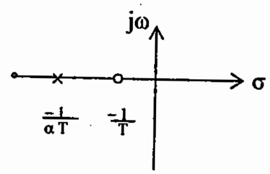
Lead compensator :

A compensator having the characteristics of a lead network is called a lead compensator. The lead compensation increases the bandwidth, which improves the speed of the response and also reduces the amount of overshoot. Lead compensation appreciably *improves the transient response.*

A lead compensator is basically *a high pass filter* and so it amplifies high frequency noise signals.

Transfer function of a lead compensator ,

$$G_c(s) = \frac{s+z_c}{s+p_c} = \frac{s+(1/T)}{s+(1/\alpha T)}$$



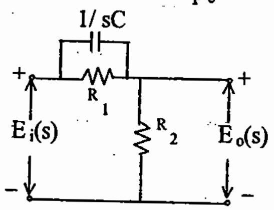


Fig : Electrical lead network

Frequency response of a lead compensator :

Consider the general form of lead compensator,

$$G_c(s) = \frac{s+(1/T)}{s+(1/\alpha T)} = \alpha\frac{(1+sT)}{(1+\alpha s T)}$$

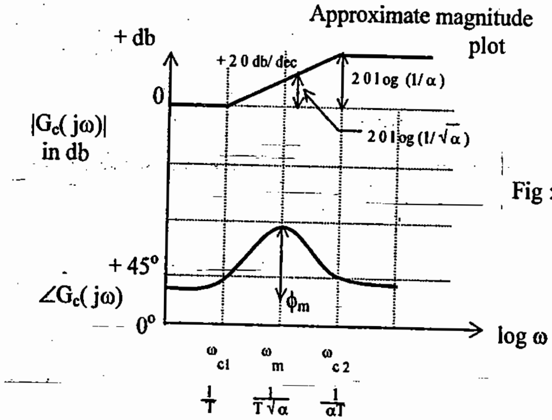


Fig : Bode plot of lead compensator.

Frequency of maximum phase lead,

$$\omega_m = \sqrt{\omega_{c1}\,\omega_{c2}} = \sqrt{(1/\alpha T).(1/T)}$$

$$= \frac{1}{T\sqrt{\alpha}}$$

The expression for maximum phase lead ϕ_m in terms of α and α in terms of ϕ_m are given below

$$\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \quad ; \quad \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Lag - Lead Compensator :

A compensator having the characteristics of lag – lead network is called a lag – lead compensator. A lag – lead compensator improves both transient and steady state response.

The transfer function of lag – lead compensator

$$G_c(s) = \frac{(s+1/T_1)}{(s+1/\beta T_1)}\;\frac{(s+1/T_2)}{(s+1/\alpha T_2)}$$

where $\beta > 1$ and $0 < \alpha < 1$

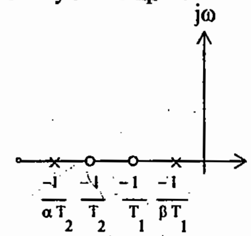


Fig : Pole – zero plot lag-lead compensator

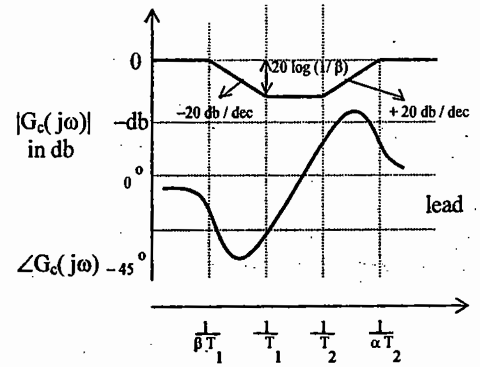


Fig : Bode plot of lag – lead compensator.

OBJECTIVE QUESTIONS

01. The lead compensator introduces
   a) Phase lead in the system
   b) Attenuation in the system
   c) Amplification in the system
   d) Initially phase lead and then phase lag in the system.

02. The lead compensator mainly
   a) improves the steady state error
   b) improves the transient response
   c) improves both steady state and transients equally
   d) None of the above

03. The lag compensator
   a) improves both steady state and transient response
   b) improves steady state only
   c) improves transients only
   d) improves steady state and reduces speed of transient response.

04. The lag-lead compensator
   a) improves steady state but reduces speed of response.
   b) improves transient response but no effect on steady state
   c) improves both steady state and transient response
   a) improves o
   b) nly the transient response

05. The lag network achieves the desired result through its
   a) attenuation property at high frequencies
   b) attenuation property at low frequencies
   c) amplification property
   d) None of the above

06. A lag network for compensation normally consists of
   a) R only
   b) R and C elements
   c) R and L elements
   d) R, L and C elements

07. The transfer function is $\frac{1+0.5s}{1+s}$ It represents a
   a) lead network
   b) lag network
   c) lag-lead network
   d) proportional controller

08. A network has a pole at $s = -1$ and a zero at $s = -2$. If this network is excited by sinusoidal input, the output
   a) leads the output
   b) lags the input
   c) is in phase with input
   d) decays exponentially to zero

Key :

1. a 2. b 3. d 4. c 5. a
6. b 7. b 8. b

JTO PREVIOUS QUESTIONS

01. The high cut off frequency function of the is $\frac{50\left(1+\frac{s}{10}\right)}{(1+s/50)}$
   (A) $10^6/2\pi$ HZ (B) $120/2\pi$ HZ
   (C) $50/2\pi$ HZ (D) $30/2\pi$ HZ

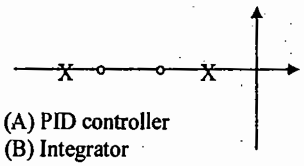
02. The transportation delays' occurring in distributed systems are determental to stability because they produce.
   (A) a phase lag
   (B) transients
   (C) attenuation
   (D) both attenuation and a phase log

03. Phase log network does which of the following?
   (A) increases system stability
   (B) Decreases bandwidth
   (C) optimal control policy
   (D) maximization control

04. Adding pole in a system causes which of the following?
   (A) Lead compensation
   (B) lead – lag – compensation
   (C) Lag compensation
   (D) None

05. Lead compensator in a feedback System
   (A) increase the system error constant to some extent
   (B) speeds up the transient response
   (C) increases the margin of stability
   (D) All of the above

06. The pole – zero plot given below is that of



   (A) PID controller
   (B) Integrator
   (C) Lag – Lead compensating network
   (D) PD controller

07. Which of the following increases the steady state accuracy?
   (A) Phase – lead compensatory
   (B) Phase – lag compensatory
   (C) Differentiator
   (D) Integrator

08. A phase lag compensating will
   (A) improve the speed of response
   (B) increase overshoot
   (C) increase relative stability
   (D) increase band width

09. The bandwidth of a control system can be increased by
   (A) phase lead compensator
   (B) phase lag compensator
   (C) phase lag – lad compensator
   (D) All of these

PREVIOUS PSU'S QUESTIONS

01. Indicate which of the following transfer function represents phase lead compensator?
   (A) $\frac{s+1}{s+5}$ (B) $\frac{6s+3}{6s+5}$
   (C) $\frac{s+5}{3s+2}$ (D) $\frac{s+5}{s^2+5s+6}$

02. Which of the following is correct expression for the transfer function of an Electrical RC Phase lag compensating network?
   (A) $\frac{RCs}{(1+RCs)}$ (B) $\frac{RCs}{(1+RCs)}$
   (C) $\frac{1}{(1+RCs)}$ (D) $\frac{1}{(1+RCs)^2}$

03. A system has the transfer function $\frac{(s+z_1)}{(1+s)}$ its gain at $\omega = 1$ rad/sec.
   (A) 1 (B) 0
   (C) – 1 (D) none of these

04. The transfer function is
   It is represents a
   (A) lead network
   (B) lag network.
   (C) lag – lead network
   (D) proportional network

05. A lag network for compensation normally consists of
   (A) R only
   (B) R and C elements
   (C) R and L elements
   (D) R, L and C elemen

06. A phase – lag compensation wi
(A) improve relative stability
(B) increase the speed of response
(C) increase bandwidth
(D) increase over shoot

07. Which one of the following Compensations is adopted for improving transient response of a negative unity feedback system?
(A) Phase lead compensation
(B) Phase lag compensation
(C) gain compensation
(D) both phase lag compensation

08. With regard to the filtering property;
the lead compensator and the lag compensator are respectively
(A) low pass and high pass filters
(B) high pass and low pass filters
(C) both high pass filters
(D) both low pass filters

09. The phase lead network function

$$G_C(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{aT}}$$

Where a < 1 would provide maximum phase – lead at a frequency of

(A) $\frac{1}{T}$ (B) $\frac{1}{aT}$
(C) $\frac{1}{T\sqrt{a}}$ (D) $\frac{1}{a\sqrt{T}}$

JTO KEYS :
(1) C (2) A (3) D (4) D (5) D
(6) C (7) D (8) C (9) A

PSU's KEYS:
(1) A (2) A (3) C (4) C (5) A
(6) A (7) D (8) A (9) A

6.2 CONTROLLERS

(1) Proportional Controller :

$$G_c(s) = K_P = \text{OLTF with controller}$$

It is used to vary the transient response of a system. Proportional controller is usually a amplifier with gain K_P.
One cannot determine the steady state response by changing K_p. Steady sate response depends on the type of the system.

(2) Integral Controller :

$$G_c(s) = K_I / s$$

It is used to decrease the steady state error by increasing the type of the system.

<u>Disadvantage</u> : Stability decreases

(3) Derivative Controller :

$$G_c(s) = K_D . s$$

It is used to increase the stability of the system. Stability of any system is increased by adding zeros.

<u>Disadvantage</u> : Steady state error increases, since type of the system decreases.

(4) Proportional + Integral (PI) Controller :

$$G_c(s) = K_P + K_I / s$$

It is used to decrease the steady state error without effecting stability. Since a pole at origin and a zero is added.

(5) Proportional + Derivative (PD) Controller :

$$G_c(s) = K_D . s + K_P$$

It is used to increase the stability without effecting steady state error. Since type is not changed and a zero is added.

(6) Proportional + Integral + Derivative (PID) Controller :

$$G_c(s) = K_P + K_I / s + K_D . s$$

$$G_c(s) = \frac{K_D . s^2 + K_P . s + K_I}{s}$$

It is used to decrease the steady state error and to increase the stability. Since pole at origin and two zeros are added. One zero compensate the pole and other zero will increase the stability.

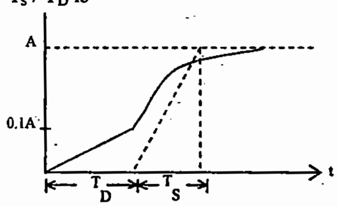
OBJECTIVE QUESTIONS

01. An integral controller is used to improve the transient response of a first-order system. If

$$G(s) = \frac{1}{1+s}$$

and the system is operated in closed-loop with unity feedback, what is the value of T_i if the integral controller transfer function is $1/T_i$ to provide damping ratio of 0.5 ?
a) 0.5 b) 2 c) 1 d) 4

02. A step response of a system is given below. T_D represents the delay due to transportation lag and T_s is the rise time of the system. As a thumb rule, the system is easily controllable if T_s / T_D is



a) less than 1 b) less than 3
c) greater than 10 d) equal to 6

03. A system has open-loop transfer given by $\frac{1}{(1+s)(1+0.5s)}$
The performance of this system

$$\frac{K(1+T_1 s)}{(1+T_2 s)}$$

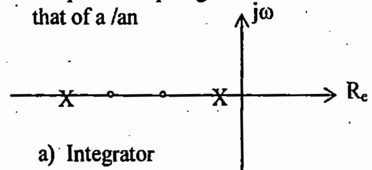
is made faster with a controller of the form. The system with controller is operated in closed-loop with unity feedback. In order to increase the speed of response
a) $T_1 = 1$
b) $T_1 = 0.5$ and $T_2 = 1$
c) $T_1 = 1$ and $T_2 = 1$
d) $T_1 = 0.5$ and $T_2 < 0.5$

04. If stability error for step input and speed of response be the criteria for design what controller would you recommend ?
a) P controller
b) PD controller
c) PI controller
d) PID controller

05. An ON–OFF controller is a
a) P controller
b) Integral controller
c) Non-linear controller
d) PID controller

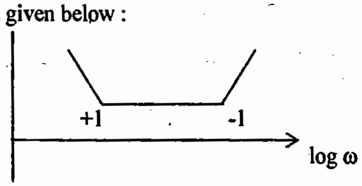
06. The term 'reset control' reefers to
a) proportional control
b) integral control
c) derivative control
d) PID controller

07. The pole-zero plot given below is that of a /an



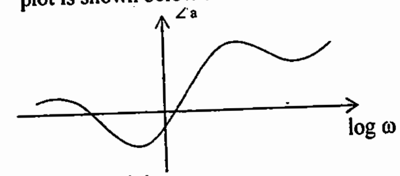
a) Integrator
b) PD controller
c) PID controller
d) Lag-lead compensating network

08. The log-magnitude plot of a system is given below :



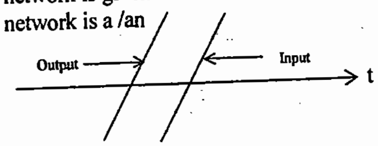
The system is an
a) Integrator
b) PID controller
c) PD controller
d) Proportional controller

09. The phase angle versus frequency plot is shown below :



The network is a
a) PID controller
b) Lag-lead network
c) PD controller d) PI controller

10. The input-output relationship of a network is given overleaf. The network is a /an



a) Integrator
b) PID controller
c) PD controller
d) Proportional controller

Key for Objective Questions :

1. c 2. c 3. d 4. d 5. c 6. b
7. d 8. b 9. b 10. c

JTO PREVIOUS QUESTION:

01. The use of PI controllers
(A) reduces oscillations
(B) results in zero steady – state error for step input
(C) lowers peak overshoot
(D) improves relative stability

02. In a PID controller, the offset has increased. The integral time constant has to be _____ so as to reduce offset
(A) increased (B) reduced to zero
(C) reduced (D) none

05. The transfer function of a rate controller is of the type
(A) 1/TS + 1 (B) 1 / TS
(C) TS (D) Kc

Previous PSU's Question

01. Derivative feedback control
(A) increases feedback time
(B) increases over shoot
(C) decreases steady state error
(D) does not affect the steady state error

02. A PD controller is used to compensate a system, Compared to The uncompensated system, the compensated system has
(A) a higher type number
(B) reduced damping
(C) higher noise amplification
(D) larger transient over shoot

03. The industrial controller having the best steady state accuracy is
(A) a derivative controller
(B) an integral controller
(C) critically damped
(D) Oscillatory

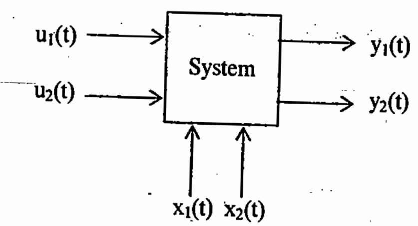
04. In industrial control systems, which one of the following methods is most commonly used in designing a system for meeting performance specifications?
(A) The transfer function is first determined and then either a lead compensation or lag compensation is implemented.
(B) The transfer function is first determined and PID controllers are implemented by thematically determining PID constants.
(C) PID controllers are implemented without the knowledge of the system parameters using Ziegler Nichols method.
(D) PID controllers are implemented using Ziegler Nichols method after determining the system transfer function

JTO KEYS : (1) B (2) A (3) C
PSU's KEYS: (1) A (2) C (3) B (4) D

Chapter: 7 STATE VARIABLE ANALYSIS

Advantages :

1) Analysis is done by considering initial conditions.
2) More accurate than transfer function
3) Analysis of multi-input-multi-output systems are less complex



Output equations

$$y_1(t) = c_{11}\, x_1(t) + c_{12}\, x_2(t) + d_{11}\, u_1(t) + d_{12}\, u_2(t)$$

$$y_2(t) = c_{21}\, x_1(t) + c_{22}\, x_2(t) + d_{21}\, u_1(t) + d_{22}\, u_2(t)$$

By representing these in matrix form,

$$\begin{pmatrix} y_1(t) \\ y_1(t) \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

$$\boxed{Y(t) = C\,X(t) + D\,U(t)}$$

State equations

$$\frac{d}{dt} x_1(t) = \dot{x}_1(t) = a_{11}\, x_1(t) + a_{12}\, x_2(t) + b_{11}\, u_1(t) + b_{12}\, u_2(t)$$

$$\frac{d}{dt} x_2(t) = \dot{x}_2(t) = a_{21}\, x_1(t) + a_{22}\, x_2(t) + b_{21}\, u_1(t) + b_{22}\, u_2(t)$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & d_{22} \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

$$\boxed{\dot{X}(t) = A\,X(t) + B\,U(t)}$$

State transition matrix :

The transition matrix is defined as a matrix that satisfies the linear homogeneous state equation.

$$\frac{d\,X(t)}{dt} = A\,X(t)$$

Let $\phi(t)$ be an $n \times n$ matrix that represents the state transition matrix; then it must satisfy the equation

$$\frac{d\,\phi(t)}{dt} = A\,\phi(t) \qquad \longrightarrow (1)$$

Let $X(0)$ denote the initial state at $t = 0$; then $\phi(t)$ is also defined by the matrix equation

$$X(t) = \phi(t)\,X(0)$$

which is the solution of the homogeneous state equation for $t \geq 0$.

One way of determining $\phi(t)$ is by taking the Laplace Transform on both sides of Eq.(1); we have

$$S\,X(s) - X(0) = A\,X(s)$$

Solving for $X(s)$ from the last equation, we get

$$X(s) = (sI - A)^{-1} X(0)$$

where it is assumed that the matrix $(sI - A)$ is nonsingular. Taking the inverse Laplace transform on both sides;

$$X(t) = x^{-1}\left[(sI - A)^{-1}\right] X(0) \quad ; \quad t \geq 0$$

Properties of the State Transition Matrix :

The state transition matrix $\phi(t)$ possesses the following properties :

1. $\phi(0) = I$ the identity matrix
2. $\phi^{-1}(t) = \phi(-t)$
3. $\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)$ for any t_0, t_1, t_2
4. $[\,\phi(t)\,]^k = \phi(kt)$

Characteristic Equation, Eigen values, and Eigen vectors :

The characteristic equation can be expressed as

$$|\,sI - A\,| = 0$$

The roots of the characteristic equation are often referred to as the eigen values of the matrix A.

The n ×1 nonzero vector P_i that satisfies the matrix equation

$$(\lambda_i I - A) P_i = 0$$

where λ_i is the eigen value of A, is called the eigenvector of A associated with the eigen value λ_i.

Controllability of linear systems :

Consider that a linear time-invariant system is described by the following dynamic equations :

$$\dot{X}(t) = A x(t) + B u(t)$$

$$Y(t) = C x(t) + D u(t)$$

where X(t) = n × 1 state vector
U(t) = r × 1 input vector
c(t) = p × 1 output vector
A = n × n coefficient matrix
B = n × r coefficient matrix
C = p × n coefficient matrix
D = p × r coefficient matrix

The state X(t) is said to be controllable at $t = t_0$ if there exists a piecewise continuous input u(t) that will drive the state to any final state $X(t_f)$ for a finite time $(t_f - t_0) \geq 0$. If every state $X(t_0)$ of the system is said to be completely state controllable or simply state controllable.

The following shows that the condition of controllability depends on the coefficient matrices A and B of the system. The theorem also gives one way of testing state controllability.

For the system to be completely state controllable, it is necessary and sufficient that the following n × nr matrix has a rank of n :

$$S = [B \;\; AB \;\; A^2B \;\ldots\ldots\; A^{n-1}B]$$

Since the matrices A and B are involved, sometimes we say that the pair [A, B] is controllable, which implies that 'S' is of rank 'n'.

Observability of Linear systems:

Given a linear time-invariant system is said to be observable if any given input u(t), there exists a finite time $t_f \geq t_0$ such that the knowledge of u(t) for $t_0 \leq t < t_f$; the matrices A, B, C, D ;

and the output c(t) for $t_0 \leq t < t_f$ are sufficient to determine $X(t_0)$. If every state of the system is observable for a finite t_f, we say that the system is completely observable, or simply observable.

The following shows that the condition of observability depends on the coefficient matrices A and D of the system. The theorem also gives one method of testing observability.

For the system to be completely observable, it is necessary and sufficient that the following n × np matrix has a rank of n :

$$V = [C^T \;\; A^TC^T \;\; (A^T)^2 C^T \;\ldots\ldots\; (A^T)^{n-1} C^T]$$

The condition is also referred to as the pair [A, C] being observable. In particular. If the system has only one output, C is an 1 × n matrix ; V is an n × n square matrix. Then the system is completely observable if V is nonsingular.

Example: Obtain the time response of the system given below :

$$\dot{X} = A X$$

where $A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$; given $x(0) = [1 \quad 1]^T$

and $y = [1 \quad -1] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Sol. : The time response is given by

$$X(t) = \phi(t)\, X(0)$$

$$\phi(t) = x^{-1} (sI - A)^{-1}$$

$$(sI - A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s \end{pmatrix}$$

$$(sI - A)^{-1} = \begin{pmatrix} s & -1 \\ 2 & s \end{pmatrix}^{-1}$$

$$(sI - A)^{-1} = \frac{\text{Adj } (sI - A)}{|sI - A|} = \begin{pmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{pmatrix}$$

Since $\phi(s) = (sI - A)^{-1}$

$$\therefore \quad \phi(s) = \begin{pmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{pmatrix}$$

∴ The state transition matrix $\phi(s)$ is

$$\phi(t) = x^{-1}\phi(s) = \begin{pmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\sqrt{2}\,t & (1/\sqrt{2})\sin\sqrt{2}\,t \\ -\sqrt{2}\sin\sqrt{2}\,t & \cos\sqrt{2}\,t \end{pmatrix}$$

$$x(t) = \phi(t)\,x(0)$$

$$\therefore \quad \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} \cos\sqrt{2}\,t & (1/\sqrt{2})\sin\sqrt{2}\,t \\ -\sqrt{2}\sin\sqrt{2}\,t & \cos\sqrt{2}\,t \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \quad x_1(t) = \cos\sqrt{2}\,t + (1/\sqrt{2})\sin\sqrt{2}\,t$$

$$x_2(t) = \sqrt{2}\sin\sqrt{2}\,t + \cos\sqrt{2}\,t$$

$$y = x_1 - x_2$$

$$\therefore \quad y = (3/\sqrt{2})\sin\sqrt{2}\,t$$

OBJECTIVE QUESTIONS

01. Given a state variable model

$$\dot{x} = A\,x + b\,u$$
$$y = c\,x + d\,u$$

Under this transformation $x = P\bar{x}$; P is a nonsingular matrix, the model becomes

$$\dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{b}\,u$$
$$y = \bar{c}\,\bar{x} + d\,u$$

a) $\bar{A} = PAP^{-1}; \bar{b} = P^{-1}b; \bar{c} = cP$
b) $\bar{A} = P^{-1}AP; \bar{b} = P^{-1}b; \bar{c} = cP$
c) $\bar{A} = P^{-1}AP; \bar{b} = Pb; \bar{c} = cP$
d) $\bar{A} = P^{-1}AP; \bar{b} = P^{-1}b; \bar{c} = cP^{-1}$

02. A state variable formation of a system is given by the equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The transfer function of the system is

a) $\frac{1}{(s+1)(s+3)}$ b) $\frac{1}{s+1}$ c) $\frac{1}{s+3}$ d) None

03. A state variable formulation of a system is given by the equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \; ; \; x_1(0) = x_2(0) = 0$$

$$y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The response y(t) to unit-step input is

a) $1 + e^{-t}$
b) $(1/3)[1 - e^{-3t}]$
c) $1 - e^{-t}$
d) None of the answers is correct

04. The Eigen values of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{pmatrix} \quad \text{are}$$

a) 0, –1, – 3 b) 0, –3, – 4 c) 0, 0, – 4 d) None

05. Given the system

$$\dot{x} = \begin{pmatrix} 0 & 0 & -20 \\ 1 & 0 & -24 \\ 0 & 1 & -9 \end{pmatrix} x + \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} u$$

$$y = [0\ 0\ 1]\,x$$

The characteristic equation of the system is

a) $s^3 + 20 s^2 + 24 s + 9 = 0$
b) $s^3 + 9 s^2 + 24 s + 20 = 0$
c) $s^3 + 24 s^2 + 9 s + 20 = 0$
d) None of the answers is correct.

06. A state variable model of a system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = [1\ \ 0] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The system is

a) controllable and observable
b) controllable but unobservable
c) observable and uncontrollable
d) uncontrollable and unobservable

07. The transfer function

$$G(s) = \mathbf{c}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{b}$$

of the system

$$\dot{x} = \mathbf{A}\,x + \mathbf{b}\,u$$

$$y = \mathbf{c}\,x + \mathbf{d}\,u$$

has pole-zero cancellation. The system is

a) uncontrollable and unobservable
b) observable but uncontrollable
c) controllable but unobservable
d) may be any one of (a), (b), and (c)

08. The transfer function

$$G(s) = \mathbf{c}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{b}$$

of the system

$$\dot{x} = \mathbf{A}\,x + \mathbf{b}\,u$$

$$y = \mathbf{c}\,x + \mathbf{d}\,u$$

has no pole-zero cancellation. The system is

a) controllable and observable
b) observable but uncontrollable
c) controllable but unobservable
d) may be any one of (a), (b), and (c)

09. Consider the system

$$A = \begin{pmatrix} 0 & -2 \\ 1 & -3 \end{pmatrix};\ b = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ c = [0\ \ 1]$$

The transfer function of the system has pole-zero cancellation. The system is

a) controllable and observable
b) uncontrollable and unobservable
c) controllable but unobservable
d) observable but uncontrollable

10. For all values of 't', the matrix exponential e^{At} is nonsingular for

a) singular A
b) nonsingular A
c) all A
d) nothing can be said, in general, about non-singularity of e^{At} for a given A.

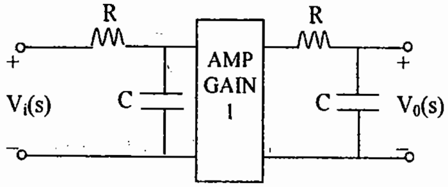
Key:

1. b 2. b 3. c 4. a 5. b 6. a 7. d 8. a 9. d 10. c

Time : 45 mts. Marks : 30

01. The transfer function of the system shown is

= RC



a) $1/(1+\tau s)$ b) $1/(1+\tau s)^2$

c) $\tau s/(1+\tau s)$ d) $\tau s/(1+\tau s)^2$

02. The performance specifications for a unity feedback control system having an open loop transfer function $G(s) = K/[s(s+1)(s+2)]$ are

i) Velocity error coefficient $K_v > 10\ sec^{-1}$

ii) Stable closed – loop operation

The value of K, satisfying above specifications, is

a) K >6 b) 6 < K < 10

c) K > 10 d) None of the above

03. A system has a complex conjugate root pair of multiplicity two or more in its characteristic equation. The impulse response of the system will be

a) a sinusoidal oscillation which decays exponentially; the system is therefore stable

b) a sinusoidal oscillation with time multiplier, the system is therefore unstable

c) a sinusoidal oscillation which rises exponentially with time, the system is therefore unstable

d) a dc term and harmonic oscillation ; the system therefore becomes limitingly stable .

04. The unit step response of a second-order linear system with zero initial state is given by $c(t) = 1+1.25\,e^{-6t}\sin(8t - \tan^{-1}1.333)$ where $t \geq 0$. The damping ratio and the undamped natural frequency of oscillation of system are respectively

a) 0.6 & 10 rad / s b) 0.6 & 12.5 rad / s

c) 0.8 & 10 rad / s d) 0.8 & 12.5 rad / s

05. Laplace Transform of a unit ramp starting at 't = a' is

a) $1/[(s+a)^2]$ b) a/s^2

c) $(e^{-as})/[(s+a)^2]$ d) $(e^{-as})/s^2$

06. The open loop transfer function of a system is

$G(s)H(s) = K/[(1+s)(1+2s+3s^2)]$.

The phase crossover frequency ω_p is

a) $\sqrt{2}$ b) 1

c) zero d) $\sqrt{3}$

07. An open loop transfer function is given by $G(s) = K(s+1)/[s(s+2)(s^2+2s+2)]$. It has

a) one zero at infinity

b) two zeroes at infinity

c) three zeroes at infinity

d) four zeros at infinity

08. Which of the following system is unstable ?

a) $K/[(1+sT_1)(1+sT_2)]$

b) $K(s+1)/[s^2(s+4)(s+5)]$, K>9

c) $K(s+2)/[(s+1)(s-3)]$, K>2

d) $K/[(Ts+1)^3]$, -1< K< 8.

09. The open loop transfer function of a unity feed back control system is given by $G(s) = K/[s(s+1)]$

If the gain is increased to infinity, then the damping ratio will tend to become

a) $1/\sqrt{2}$ b) 1

c) 0 d) ∞

10. The characteristic equation of a closed – loop system is given by

$s^4 + 6s^3 + 11s^2 + 6s + K = 0$.

Stable closed – loop behaviour can be ensured when gain K is such that

a) 0 < K < 10 b) K > 10

c) $-\infty \leq K > \infty$ d) $0 < K \leq 20$

11. The maximum phase shift that can be obtained by using a lead compensator with transfer function

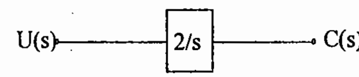
$G_c(s) = 4(1+0.15s)/(1+0.05s)$

is equal to

a) 15^0 b) 30^0

c) 45^0 d) 60^0

12. Consider a system shown. If the system is disturbed so that c(0) = 1, then c(t) for a unit step input will be



a) 1 + t b) 1 – t

c) 1 + 2t d) 1 – 2t

13. The closed loop transfer function of a control system is given by 1/(1+s). For the input r(t) = sint the steady state value of c(t) is equal to

a) $(1/\sqrt{2})$ cos t b) 1

c) $(1/\sqrt{2})$ sin t d) $(1/\sqrt{2})\sin[t-(\pi/4)]$

14. The impulse response of an initially relaxed linear system is $e^{-2t}u(t)$. To produce a response of $te^{-2t}u(t)$, the input must be equal to

a) $2e^{-t}u(t)$ b) $\frac{1}{2}e^{-2t}u(t)$

c) $e^{-2t}u(t)$ d) $e^{-t}u(t)$

15. The closed loop transfer function of a control system is given by

$C(s)/R(s) = 2(s-1)/[(s+2)(s+1)]$

For a unit step input the output is

a) $-3e^{-2t} + 4e^{-t} - 1$ b) $-3e^{-2t} - 4e^{-t} + 1$

c) Zero d) Infinity

16. The Laplace transformation of $f(t)$ is F(s). Given $F(s) = K/[(s+2)(s^2+4)]$, the final value of $f(t)$ is

a) Infinity b) zero

c) one d) none of the above

17. For M > 1, the constant M – circles corresponding to the magnitude (M) of the closed loop transfer function of a linear system lie in the G – plane and to the ______

a) right of the M = 1 line

b) left of the M = 1 line

c) upper side of the M = +j1 line

c) lower side of the M = -j 1 line

18. The position and acceleration error coefficients for the open loop transfer function

$G(s) = K/[s^2(s+10)(s+100)]$ are respectively

a)zero and infinity

b)infinity and zero

c) (K / 100) and zero

d) infinity and (K / 1000)

19. The position and velocity error coefficients for the system of transfer function

$G(s) = K/[(1+0.1s)(1+2s)]$ are respectively

a) K & zero

b) zero & infinity

c) [1/(K+1)] &zero

d) [1/(K+1)] & infinity

20. The transfer function of system is

$F(s) = 10(1+0.2s)/[(2+0.5s)]$.

The phase shift at ω = 0 and ω = ∞ will be

a) 90^0 and 0^0 b) -180^0 and 180^0

c) -90^0 and 70^0 d) None of the above

21. A system has the transfer function $[(1-s)/(1+s)]$. It is known as

a) low pass system b) high pass system

c) all pass system d) none of the above

22. The transfer function of a control system is given as
$T(s) = K / [(s^2 + 2s + K)]$,
where K is the gain of the system in radian/ amp. For this system to be critically damped, the value of K should be

(a) 1 (b) 2 (c) 3 (d) 4

23. A second order differential equation is given by. The natural frequency in rad/sec and damping ratio are respectively

(a) 1,5 (b) 5, 7
(c) 1, $\sqrt{7}$ (d) $\sqrt{7}$

24. The transfer function of a system is $10 / (1 + s)$. When operated as a unity feedback system, the steady state error to a unit step input will be

(a) zero (b) 1 / 11
(c) 10 (d) Infinity

25. By Routh's stability criterion, it is possible to find the roots in RHP of the vertical line passing through (a, o) by substituting in the characteristic polynomial

(a) $s = (\lambda + a)$ (a) $s = (\lambda - a)$
(c) $s = (\lambda + 1 + a)$ (d) $s = (\lambda + 1 - a)$

26. If the gain of the open loop system is doubled, the gain margin

(a) is not affected (b) gets doubled
(c) becomes half
(d) becomes one – fourth

27. Given $G(s) = (1-s) / [s(s+2)]$. The system with this transfer function is operated in a closed loop with unity feedback. The closed loop system is

(a) stable (b) unstable
(c) marginally stable
(d) conditionally stable

28. The transfer function of transportation lag is e^{-sT}. If the lag is small as compared with the time constant of the system, it can be approximated by
(a) sT (b) 1 + sT
(c) 1 – sT (d) 1 / (1 + sT)

29. The characteristic equation of a certain system is $s^5 + 4s^4 + 2s^3 + 8s^2 + s + 4 = 0$. This system is

(a) stable (b) marginally Stable
(c) unstable (d) critical damped

30. Phase margin of a system is used to specify

(a) relative stability
(b) absolute stability
(c) time response
(d) frequency response

KEY FOR GRAND TEST

01.b	02.d	03.b	04.a	05.d
06.b	07.c	08.b	09.c	10.a
11.b	12.c	13.d	14.c	15.a
16.d	17.b	18.d	19.a	20.d
21.c	22.d	23.d	24.b	25.a
26.c	27.a	28.c	29.c	30.a

"All The Best"

OBJECTIVE QUESTIONS : REVISION AND PRACTICE SET

One Mark Questions

01. For a second-order system with the closed-loop transfer function

$T(s) = \dfrac{9}{s^2 + 4s + 9}$ the settling time for 2-percent band, in seconds, is **GATE-1999**

(a) 1.5 (b) 2.0 (c) 3.0 (d) 4.0

02. The gain margin (in dB) of a system having the loop transfer function

$G(s)H(s) = \dfrac{\sqrt{2}}{s(s+1)}$ is GATE-1999

(a) 0 (b) 3 (c) 6 (d) ∞

03. The system mode described by the state equations

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$, $y = [1 \ \ 1] x$ is **GATE-1999**

(a) controllable and observable
(b) controllable, but not observable
(c) observable, but not controllable
(d) neither controllable nor observable

04. The phase margin (in degrees) of a system having the loop transfer function

$G(s)H(s) = \dfrac{2\sqrt{3}}{s(s+1)}$ is **GATE-1999**

(a) 45 (b) –30
(c) 60 (d) 30

05. An amplifier with resistive negative feedback has two left half plane poles in its open-loop transfer function. The amplifier **GATE-2000**
(a) will always be unstable at high frequency
(b) will be stable for all frequency
(c) may be unstable, depending on the feedback factor
(d) will oscillate at low frequency

06. A system described by the transfer function $H(s) = \dfrac{1}{s^3 + \alpha s^2 + ks + 3}$ is stable.
The constraints on α and k are, **GATE-2000**
(a) $\alpha > 0, \alpha k < 3$ (b) $\alpha > 0, \alpha k > 3$
(c) $\alpha < 0, \alpha k > 0$ (d) $\alpha > 0, \alpha k < 0$

07. The equivalent of the block diagram in Fig-1 is given in **GATE-2001**

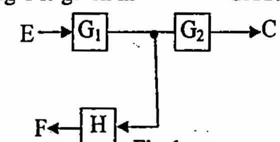


Fig-1

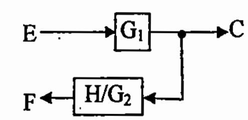
(a)


Fig-A

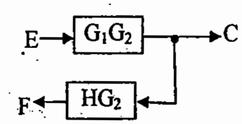
(b)


Fig-B

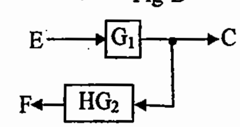
(c)


Fig-C

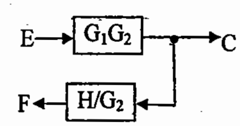
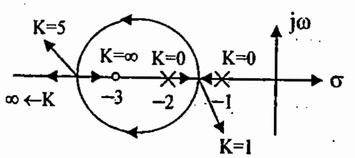
(d)


Fig-D

(a) Fig-A (b) Fig-B
(c) Fig-C (d) Fig-D

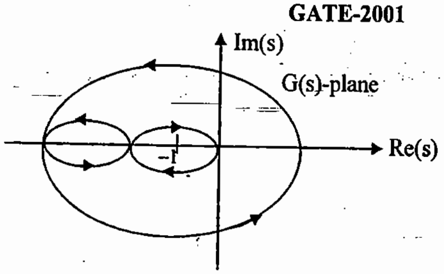
08. If the characteristic equation of a closed-loop system is $s^2+2s+2 = 0$, then the system is **GATE-2001**
(a) overdamped
(b) critically damped
(c) underdamped
(d) undamped

09. The root-locus diagram for a closed-loop feedback system is shown in figure. The system is overdamped. **GATE-2001**



(a) only if $0 \le K \le 1$
(b) only if $1 < K < 5$
(c) only if $K > 5$
(d) if $0 \le K < 1$ or $K > 5$

10. The Nyquist plot for the open-loop transfer function G(s) of a unity negative feedback system is shown in figure. If G(s) has no pole in the right-half of s-plane, the number of roots of the system characteristics equation in the right-half of s-plane is **GATE-2001**



(a) 0 (b) 1 (c) 2 (d) 3

11. An electrical system and its signal-flow graph representation are shown in figure(a) and (b) respectively. The values of G_2 and H, respectively, are **GATE-2001**

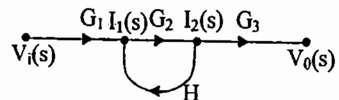


Figure (b)

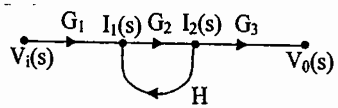


Figure (b)

(a) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(b) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

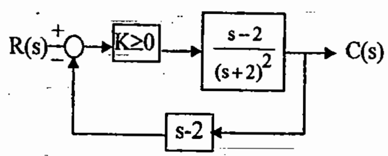
(c) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

(d) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

12. The open-loop DC gain of a unity negative feedback system with closed-loop transfer function $\dfrac{s+4}{s^2+7s+13}$ is **GATE-2001**

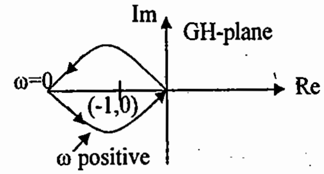
(a) $\dfrac{4}{13}$ (b) $\dfrac{4}{9}$ (c) 4 (d) 13

13. The feedback control system in the figure is stable. **GATE-2001**



(a) for all $K \ge 0$ (b) only if $K \ge 1$
(c) only if $0 \le K < 1$ (d) only if $0 \le K \le 1$

14. Figure shows the Nyquist plot of the open-loop transfer function G(s)H(s) of a system. If G(s)H(s) has one right-hand pole, the closed-loop system is **GATE-2003**



(a) always stable
(b) unstable with one closed-loop right hand pole
(c) unstable with two closed-loop right hand poles
(d) unstable with three closed-loop right hand poles

15. A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has **GATE-2003**
(a) a higher type number
(b) reduced damping
(c) higher noise amplification
(d) larger transient overshot

16. The gain margin for the system with open-loop transfer function

$G(s)H(s) = \dfrac{2(1+s)}{s^2}$, is **GATE 2004**

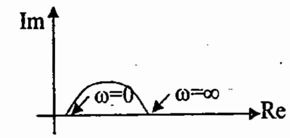
(a) ∞ (b) 0 (c) 1 (d) −∞

17. Given $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$, the point of intersection of the asymptotes of the root loci with the real axis is **GATE 2004**
(a) −4 (b) 1.33
(c) −1.33 (d) 4

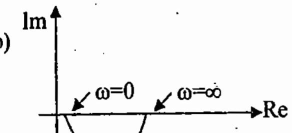
18. A linear system is equivalently represented by two sets of state equations.
$\dot{X} = AX+BU$ and $\dot{W} = CW+DU$
The eigen values of the representations are also computed as [λ] and [μ]. Which one of the following statements is true? **GATE 2005**
(a) $[\lambda] = [\mu]$ and $X = W$
(b) $[\lambda] = [\mu]$ and $X \ne W$
(c) $[\lambda] \ne [\mu]$ and $X = W$
(d) $[\lambda] \ne [\mu]$ and $X \ne W$

19. Which one of the following polar diagrams corresponds to a lag network? **GATE 2005**

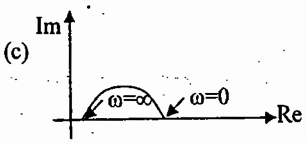
(a)



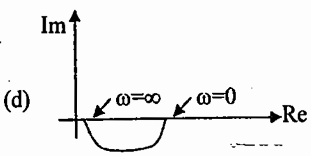
(b)



(c)



(d)



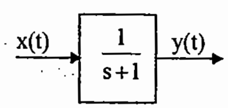
20. Despite the presence of negative feedback, control systems still have problems of instability because the **GATE 2005**
(a) components used have nonlinearities
(b) dynamic equations of the subsystems are not known exactly
(c) mathematical analysis involves approximations
(d) system has large negative phase angle at high frequencies.

21. The open-loop transfer function of a unity-gain feedback control system is given by **GATE 2006**

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by
(a) 0 (b) 1 (c) 20 (d) ∞

22. In the system shown below, $x(t) = (\sin t)u(t)$. In steady-state the response $y(t)$ will be **GATE 2006**



(a) $\frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$ (b) $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$
(c) $\frac{1}{\sqrt{2}}e^{-t}\sin t$ (d) $\sin t - \cos t$

23. If the closed-loop transfer function of a control system is given as

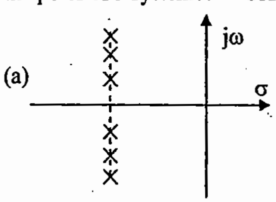
$$T(s) = \frac{s-5}{(s+2)(s+3)}$$, then it is **GATE 2007**

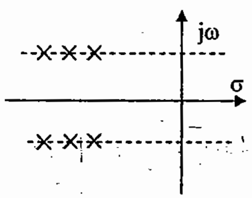
(a) an unstable system
(b) an uncontrollable system
(c) a minimum phase system
(d) a non-minimum phase system

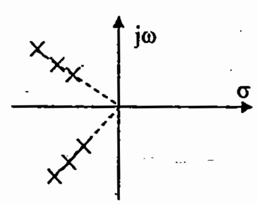
24. If the Laplace transform of a signal y(t) is $Y(s) = \frac{1}{s(s-1)}$, then its final value is **GATE-2007**
(a) -1 (b) 0
(c) 1 (d) unbounded

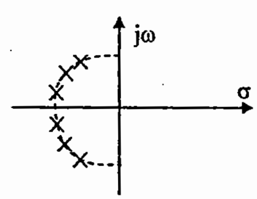
25. The pole-zero plot given below corresponds to a **GATE 2008**
(a) Low pass filter
(b) High pass filter
(c) Band pass filter
(d) notch filter

26. Step responses of a set of three second-order underdamped systems all have the same percentage overshoot. Which of the following diagrams represents the poles of 3 systems? **GATE 2008**

(a)


(b)


(c)


(d)


27. Which of the following points is NOT on the root locus of a system with the open-loop transfer function **GATE-2002**

$$G(s)H(s) = \frac{k}{s(s+1)(s+3)}$$?

(a) $s = -j\sqrt{3}$ (b) $s = -1.5$
(c) $s = -3$ (d) $s = -\infty$

28. The phase margin of a system with the open-loop transfer function **GATE-2002**

$$G(s)H(s) = \frac{(1-s)}{(1+s)(2+s)}$$

(a) 0^0 (b) 63.4^0
(c) 90^0 (d) ∞

29. The transfer function Y(s)/U(s) of a system described by the state equations $\dot{x}(t) = -2x(t) + 2u(t)$ and $y(t) = 0.5x(t)$ is **GATE-2002**
(a) 0.5/(s–2) (b) 1/(s–2)
(c) 0.5/(s+2) (d) 1/(s+2)

30. Consider the system $\frac{dx}{dt} = Ax + Bu$

with $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix}$

where p and q are arbitrary real numbers. Which of the following statements about the controllability of the system is true?
(a) The system is completely state controllable for any nonzero values of p and q.
(b) Only p =0 and q=0 result in controllability.
(c) The system is uncontrollable for all values of p and q.
(d) We cannot conclude about controllability from the given data

Two Mark Questions

01. If the closed-loop transfer function T(s) of a unity negative feedback system is given by

$$T(s)=\frac{a_{n-1}s+a_n}{s^n+a_1s^{n-1}+....+a_{n-1}s+a_n}$$

then the steady state error for a unit ramp input is **GATE-1999**

(a) $\frac{a_n}{a_{n-1}}$ (b) $\frac{a_n}{a_{n-2}}$

(c) $\frac{a_{n-1}}{a_{n-2}}$ (d) zero

02. Consider the points $s_1=-3+j4$ and $s_2=-3-j2$ in the s-plane. Then, for a system with the open-loop transfer function

$$G(s)H(s)=\frac{K}{(s+1)^4}$$ **GATE-1999**

(a) s_1 is on the root locus, but not s_2
(b) s_2 is on the root locus, but not s_1
(c) both s_1 and s_2 are on the root locus
(d) neither s_1 nor s_2 is on the root locus

03. For the system described by the state equation

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\0.5&1&2\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

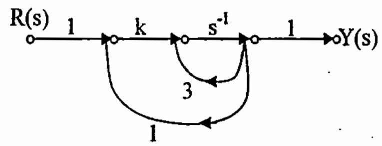
if the control signal u is given by $u=[-0.5\ -3\ -5]x+v$, then the eigen values of the closed-loop system will be **GATE-1999**

(a) 0, –1, –2 (b) 0, –1, –3
(c) –1, –1, –2 (d) 0, –1, –1

04. Consider a system with the transfer function $G(s)=\frac{s+6}{ks^2+s+6}$. Its damping ratio will be 0.5 when the value of k is

(a) 2/6 (b) 3 (c) 1/6 (d) 6

05. The system shown in the figure remains stable when **GATE-2002**



(a) k < –1 (b) –1 < k < 1
(c) 1 < k < 3 (d) k > 3

06. The transfer function of a system is

$$G(s)=\frac{100}{(s+1)(s+100)}$$. For a unit-step input to the system the approximate settling time for 2% criterion is

(a) 100 sec (b) 4 sec
(c) 1 sec (d) 0.01 sec

07. The characteristic polynomial of a system $q(s)=2s^5+s^4+4s^3+2s^2+2s+1$. The system is **GATE-2002**

(a) stable (b) marginally stable
(c) unstable (d) ocillatory

08. The system with the open loop transfer function $G(s)H(s)=\frac{1}{s(s^2+s+1)}$ has a gain margin of **GATE-2002**

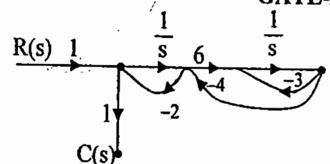
(a) –6 dB (b) 0 dB
(c) 3.5 dB (d) 6 dB

09. The root locus of the system

$$G(s)H(s)=\frac{K}{s(s+2)(s+3)}$$ has the break-away point located at **GATE-2003**

(a) (–0.5, 0) (b) (–2.548, 0)
(c) (–4, 0) (d) (–0.748, 0)

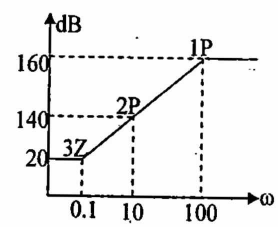
10. The signal flow graph of a system is shown in figure. The transfer function $\frac{C(s)}{R(s)}$ of the system is **GATE-2003**



(a) $\frac{6}{s^2+29s+6}$ (b) $\frac{6s}{s^2+29s+6}$

(c) $\frac{s(s+2)}{s^2+29s+6}$ (d) $\frac{s(s+27)}{s^2+29s+6}$

11. The approximate Bode magnitude plot of a minimum-phase system is shown in figure. The transfer function of the system is **GATE-2003**



(a) $10^8\frac{(s+0.1)^3}{(s+10)^2(s+100)}$

(b) $10^7\frac{(s+0.1)^3}{(s+10)(s+100)}$

(c) $10^8\frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(d) $10^9\frac{(s+0.1)^3}{(s+10)(s+100)^2}$

12. A second-order system has the transfer function $\frac{C(s)}{R(s)}=\frac{4}{s^2+4s+4}$. With r(t) as the unit-step function, the response c(t) of the system is represented by **GATE-2003**

(a)

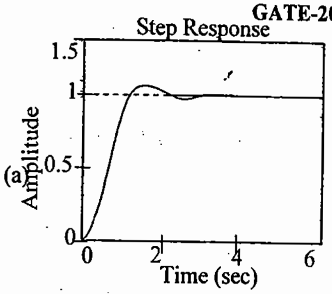


Fig-1

(b)

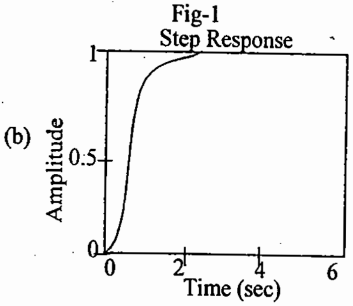


Fig-2

(c)

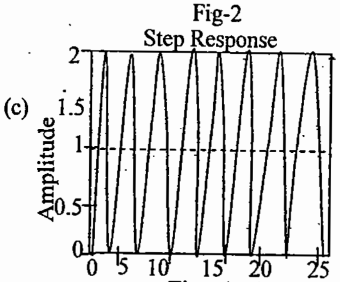


Fig-3

(d)

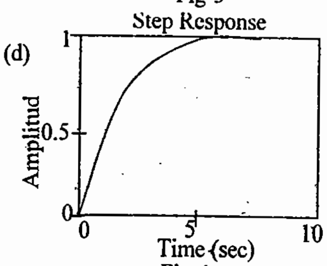


Fig-4

(a) Fig-1 (b) Fig-2

(c) Fig-3 (d) Fig-4

13. The gain margin and the phase margin of a feedback system with GATE-2003

$$G(s)H(s)=\frac{s}{(s+100)^3} \text{ are}$$

(a) 0 dB, 0^0 (b) ∞, ∞

(c) $\infty, 0^0$ (d) 88.5 dB, ∞

14. The zero-input response of a system given by the state-space equation

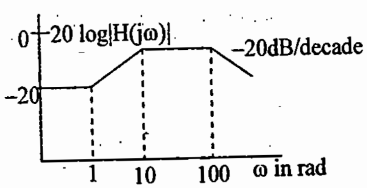
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \text{ and } \begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$$

is GATE-2003

(a) $\begin{bmatrix}te^t\\ t\end{bmatrix}$ (b) $\begin{bmatrix}e^t\\ t\end{bmatrix}$

(c) $\begin{bmatrix}e^t\\ te^t\end{bmatrix}$ (d) $\begin{bmatrix}t\\ te^t\end{bmatrix}$

15. Consider the Bode magnitude plot shown in figure. The transfer function H(s) is GATE-2004



(a) $\frac{(s+10)}{(s+1)(s+100)}$ (b) $\frac{10(s+1)}{(s+10)(s+100)}$

(c) $\frac{10^2(s+1)}{(s+10)(s+100)}$ (d) $\frac{10^3(s+100)}{(s+1)(s+10)}$

16. A causal system having the transfer function $H(s)=\frac{1}{s+2}$ is excited with $10u(t)$. The time at which the output reaches 99% of its steady state value is GATE-2004

(a) 2.7 sec (b) 2.5 sec

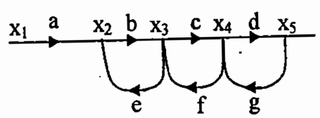
(c) 2.3 sec (d) 2.1 sec

17. A system has poles at 0.01 Hz, 1Hz and 80 Hz; zeros at 5Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is GATE-2004

(a) -90^0 (b) 0^0

(c) 90^0 (d) -180^0

18. Consider the signal flow graph shown in figure. The gain $\frac{x_5}{x_1}$ is GATE-2004



(a) $\frac{1-(be+cf+dg)}{abcd}$ (b) $\frac{bedg}{1-(be+cf+dg)}$

(c) $\frac{abcd}{1-(be+cf+dg)+bedg}$

(d) $\frac{1-(be+cf+dg)+bedg}{abcd}$

19. The open-loop transfer function of a unity feedback system is

$$G(s)=\frac{K}{s(s^2+s+2)(s+3)}$$

The range of K for which the system is stable is GATE-2004

(a) $\frac{21}{4}>K>0$ (b) $13>K>0$

(b) $\frac{21}{4}<K<\infty$ (d) $-6<K<\infty$

20. For the polynomial $P(s)=s^5+s^4+2s^3+2s^2+3s+15$, the number of roots which lie in the right half of the s-plane is GATE-2004

(a) 4 (b) 2 (c) 3 (d) 1

21. The state variable equations of a system are:

$\dot{x}_1=-3x_1-x_2+u$

$\dot{x}_2=2x_1$, $y=x_1+u$

The system is GATE-2004

(a) controllable but not observable
(b) observable but not controllable
(c) neither controllable nor observable
(d) controllable and observable

22. A system described by the following differential equation

$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=x(t)$ is initially at rest. For input $x(t)=2u(t)$, the output $y(t)$ is GATE-2004

(a) $(1-2e^{-t}+e^{-2t})u(t)$

(b) $(1+2e^{-t}-2e^{-2t})u(t)$

(c) $(0.5+e^{-t}+1.5e^{-2t})u(t)$

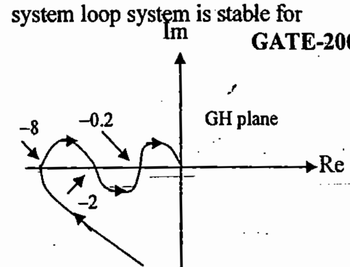
(d) $(0.5+2e^{-t}+2e^{-2t})u(t)$

23. Given $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, the state transition matrix e^{At} is given by GATE-2004

(a) $\begin{bmatrix}0 & e^{-t}\\ e^{-t} & 0\end{bmatrix}$ (b) $\begin{bmatrix}e^t & 0\\ 0 & e^t\end{bmatrix}$

(c) $\begin{bmatrix}e^{-t} & 0\\ 0 & e^{-t}\end{bmatrix}$ (d) $\begin{bmatrix}0 & e^t\\ e^t & 0\end{bmatrix}$

24. The polar diagram of a conditionally stable system for open loop gain K = 1 is shown in figure. The open loop transfer function of the system is known to be stable. The closed loop system loop system is stable for GATE-2005



(a) $K<5$ and $\frac{1}{2}<K<\frac{1}{8}$

(b) $K<\frac{1}{8}$ and $\frac{1}{2}<K<5$

(c) $K<\frac{1}{8}$ and $5<K$

(d) $K>\frac{1}{8}$ and $K<5$

25. In the derivation of expression for peak percent overshoot,

$$M_p=\exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)\times 100\%,$$

which one of the following conditions is NOT required? GATE-2005

(a) System is linear and time invariant
(b) The system transfer function has a pair of complex conjugate poles and no zeroes.
(c) There is no transportation delay in the system.
(d) The system has zero initial conditions.

26. A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively GATE-2005

(a) 1 and 20 (b) 0 and 20

(c) 0 and $\frac{1}{20}$ (d) 1 and $\frac{1}{20}$

27. A double integrator plant,

$G(s)=\frac{K}{s^2}, H(s)=1$ is to be compensated to achieve the damping ratio $\xi=0.5$, and an undamped natural frequency, ω_n=5 rad/s. Which one of the following compensator $G_c(s)$ will be suitable? **GATE-2005**

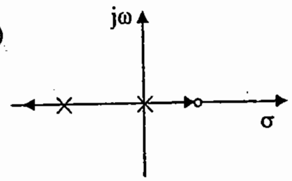
(a) $\frac{s+3}{s+9.9}$ (b) $\frac{s+9.9}{s+3}$

(b) $\frac{s-6}{s+8.33}$ (d) $\frac{s+6}{s}$

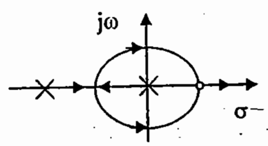
28. An unity feedback system is given as,

$G(s)=\frac{K(1-s)}{s(s+3)}$. Indicate the correct root locus diagram **GATE-2005**

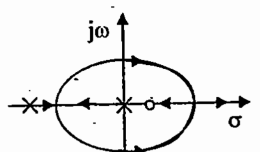
(a)



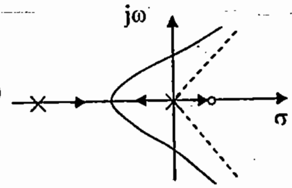
(b)



(c)



(d)



Common Data for Questions 29 and 30.
The open loop transfer function of a unity feedback system is given by

$$G(s)=\frac{3e^{-2s}}{s(s+2)}$$

29. The gain and phase crossover frequencies in rad/sec are, respectively **GATE-2005**

(a) 0.632 and 1.26
(b) 0.632 and 0.485
(c) 0.485 and 0.632
(d) 1.26 and 0.632

30. Based on the above results, the gain and phase margins of the system will be **GATE-2005**

(a) –7.09 and 87.5^0
(b) 7.09 and 87.5^0
(c) 7.09 dB and -87.5^0
(d) –7.09 dB and -87.5^0

31. Consider two transfer functions

$G_1(s)=\frac{1}{s^2+as+b}$ and $G_2(s)=\frac{s}{s^2+as+b}$

The 3-dB bandwidths of their frequency responses are, respectively **GATE-2006**

(a) $\sqrt{a^2-4b}$, $\sqrt{a^2+4b}$
(b) $\sqrt{a^2+4b}$, $\sqrt{a^2-4b}$
(c) $\sqrt{a^2-4b}$, $\sqrt{a^2-4b}$
(d) $\sqrt{a^2+4b}$, $\sqrt{a^2+4b}$

32. The unit-step response of a system starting from rest is given by
$c(t)=1-e^{-2t}$ for $t\geq 0$
The transfer function of the system is **GATE-2006**

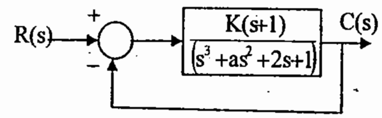
(a) $\frac{1}{1+2s}$ (b) $\frac{2}{2+s}$

(c) $\frac{1}{2+s}$ (d) $\frac{2s}{1+2s}$

33. The Nyquist plot of G(jω) H(jω) for a closed loop control system, passes through (–1,j0) point in the GH plane. The gain margin of the system in dB is equal to **GATE-2006**

(a) infinite
(b) greater than zero
(c) less than zero
(d) zero

34. The positive values of "K" and "a" so that the system shown in the figure below oscillates at a frequency of 2 rad/sec respectively are **GATE-2006**



(a) 1, 0.75 (b) 2, 0.75
(c) 1, 1 (d) 2, 2

35. The unit impulse response of a system is $h(t)=e^{-t}$, $t\geq0$
For this system, the steady-state value of the output for unit step input is equal to **GATE-2006**
(a) –1 (b) 0 (c) 1 (d) ∞

36. The transfer function of a phase-lead compensator is given by

$G_c(s)=\frac{1+3Ts}{1+Ts}$ where $T>0$
The maximum phase-shift provided by such a compensator is **GATE-2006**
(a) π/2 (b) π/3
(c) π/4 (d) π/6

37. A linear system is described by the following state equation

$\dot{X}(t)=AX(t)+BU(t), A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

The state-transition matrix of the system is

(a) $\begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$ (b) $\begin{bmatrix}-\cos t & \sin t\\ -\sin t & -\cos t\end{bmatrix}$

(c) $\begin{bmatrix}-\cos t & -\sin t\\ -\sin t & \cos t\end{bmatrix}$ (d) $\begin{bmatrix}\cos t & -\sin t\\ \cos t & \sin t\end{bmatrix}$

Common Data for Questions 38 and 39.
Consider a unity-gain feedback control system whose open-loop transfer function is $G(s)=\frac{as+1}{s^2}$

38. The value of "a" so that the system has a phase-margin equal to π/4 is approximately equal to **GATE-2006**
(a) 2.40 (b) 1.40
(c) 0.84 (d) 0.74

39. With the value of "a" set for phase-margin of π/4, the value of unit-impulse response of the open-loop system at t=1 second is equal to **GATE-2006**
(a) 3.40 (b) 2.40
(c) 1.84 (d) 1.74

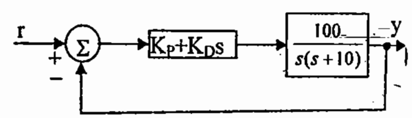
40. The frequency response of a linear, time-invariant system is given by

$H(f)=\frac{5}{1+j10\pi f}$. The step response of the system is **GATE-2007**

(a) $5(1-e^{-5t})u(t)$ (b) $5\left(1-e^{-\frac{t}{5}}\right)u(t)$

(c) $\frac{1}{5}\left(1-e^{-5t}\right)u(t)$ (d) $\frac{1}{5}\left(1-e^{-\frac{t}{5}}\right)u(t)$

41. A control system with a PD controller is shown in the figure. If the velocity error constant K_V=1000 and the damping ratio ζ=0.5, then the values of K_P and K_D are **GATE-2007**



(a) K_P=100, K_D=0.09
(b) K_P=100, K_D=0.9
(c) K_P=10, K_D=0.09
(d) K_P=10, K_D=0.9

42. The transfer function of a plant is

$T(s)=\frac{5}{(s+5)(s^2+s+1)}$. The second-order approximation of T(s) using dominant pole concept is **GATE-2007**

(a) $\frac{1}{(s+5)(s+1)}$ (b) $\frac{5}{(s+5)(s+1)}$

(c) $\frac{5}{s^2+s+1}$ (d) $\frac{1}{s^2+s+1}$

43. The open-loop transfer function of a plant is given as $G(s)=\frac{1}{s^2+s+1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is **GATE-2007**

(a) $\frac{10(s-1)}{s+2}$ (b) $\frac{10(s+4)}{s+2}$

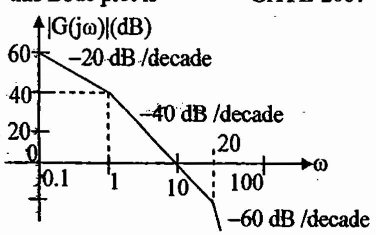
(b) $\frac{10(s+2)}{s+10}$ (d) $\frac{10(s+2)}{s+10}$

44. A unity feedback control system has an open-loop transfer function

$G(s)=\frac{K}{s(s^2+7s+12)}$. The gain K for which s=−1+j1 will lie on the root locus of this system is **GATE-2007**

(a) 4 (b) 5.5 (c) 6.5 (d) 10

45. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function G(s) corresponding to this Bode plot is **GATE-2007**



(a) $\frac{1}{(s+1)(s+20)}$ (b) $\frac{1}{s(s+1)(s+20)}$

(c) $\frac{100}{s(s+1)(s+20)}$ (d) $\frac{100}{s(s+1)(1+0.05s)}$

46. The state space representation of a separately excited DC servo motor dynamics is given as **GATE-2007**

$$\begin{bmatrix}\frac{d\omega}{dt}\\ \frac{di_a}{dt}\end{bmatrix}=\begin{bmatrix}-1 & 1\\ -1 & -10\end{bmatrix}\begin{bmatrix}\omega\\ i_a\end{bmatrix}+\begin{bmatrix}0\\ 10\end{bmatrix}u$$

where ω is the speed of the motor, i_a is the armature current and u is the armature voltage. The transfer function $\frac{\omega(s)}{U(s)}$ of the motor is **GATE-2007**

(a) $\frac{10}{s^2+11s+11}$ (b) $\frac{1}{s^2+11s+11}$

(c) $\frac{10s+10}{s^2+11s+11}$ (d) $\frac{1}{s^2+s+1}$

Statement for Linked Answer Questions 47 & 48:

Consider a linear system whose state space representation is $\dot{x}(t)=Ax(t)$. If the initial state vector of the system is $x(0)=\begin{bmatrix}1\\ -2\end{bmatrix}$, then the system response is

$x(t)=\begin{bmatrix}e^{-2t}\\ -2e^{-2t}\end{bmatrix}$. If the initial state vector of the system changes to $x(0)=\begin{bmatrix}1\\ -1\end{bmatrix}$, then the system response becomes

$x(t)=\begin{bmatrix}e^{-t}\\ -e^{-t}\end{bmatrix}$

47. The eigenvalue and eigenvector pairs(λ_i, v_i) for the system are **GATE-2007**

(a) $\left[-1,\begin{bmatrix}1\\ -1\end{bmatrix}\right]$ and $\left[-2,\begin{bmatrix}1\\ -2\end{bmatrix}\right]$

(b) $\left[-2,\begin{bmatrix}1\\ -1\end{bmatrix}\right]$ and $\left[-1,\begin{bmatrix}1\\ -2\end{bmatrix}\right]$

(c) $\left[-1,\begin{bmatrix}1\\ -1\end{bmatrix}\right]$ and $\left[2,\begin{bmatrix}1\\ -2\end{bmatrix}\right]$

(d) $\left[-2,\begin{bmatrix}1\\ -1\end{bmatrix}\right]$ and $\left[1,\begin{bmatrix}1\\ -2\end{bmatrix}\right]$

48. The system matrix A is **GATE-2007**

(a) $\begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$

(c) $\begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$

49. A linear, time-invariant, causal continuous time system has a rational transfer function with simple poles at s = −2 and s = −4, and one simple zero at s = −1. A unit step u(t) is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of this system is **GATE-2008**

(a) [exp(−2t) + exp(−4t)] u(t)

(b) [−4exp(−2t) +12exp(−4t)−exp(−t)]u(t)

(c) [−4exp(−2t) + 12exp(−4t)]u(t)

(d) [−0.5exp(−2t) + 1.5exp(−4t)]u(t)

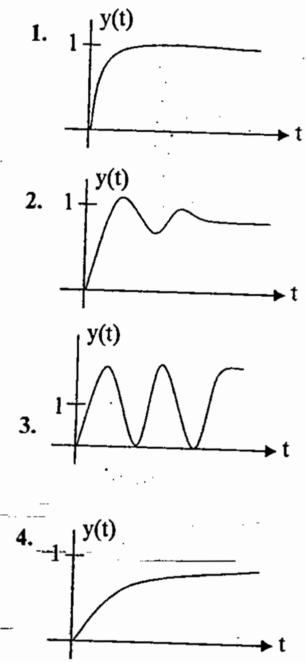
50. **Group I** lists a set of four transfer functions. **Group II** gives a list of possible step responses y(t). Match the step responses with the corresponding transfer functions. **GATE-2008**

Group I

$P=\frac{25}{s^2+25}$ $Q=\frac{36}{s^2+20s+36}$

$R=\frac{36}{s^2+12s+36}$ $S=\frac{49}{s^2+7s+49}$

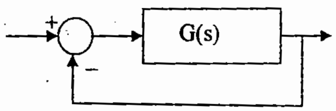
Group II



(a) P-3, Q-1, R-4, S-2
(b) P-3, Q-2, R-4, S-1
(c) P-2, Q-1, R-4, S-3
(d) P-3, Q-4, R-1, S-2

51. A certain system has transfer function

$$G(s)=\frac{s+8}{s^2+\alpha s-4}$$, where α is a parameter. Consider the standard negative unity feedback configuration as shown below.



Which of the following statements is true? **GATE-2008**

(a) The closed loop system is never stable for any value of α
(b) For some positive values of α, the closed loop system is stable, but not for all positive values
(c) For all positive values of α, the closed system is stable
(d) The closed loop system is stable for all values of α, both positive and negative

52. The number of open right half plane poles of

$$G(s)=\frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$$ is

GATE-2008

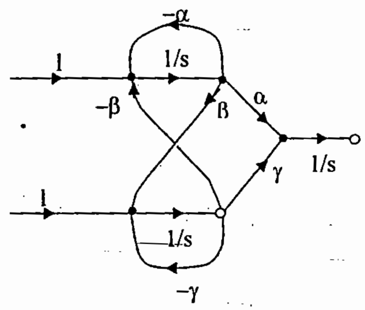
(a) 0 (b) 1 (c) 2 (d) 3

53. The magnitude of frequency response of an underdamped second order system is 5 at 0 rad/sec and peaks to $\frac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is **GATE-2008**

(a) $\frac{500}{s^2+10s+100}$ (b) $\frac{375}{s^2+5s+75}$

(b) $\frac{720}{s^2+12s+144}$ (d) $\frac{1125}{s^2+25s+225}$

54. A signal flow graph of a system is given below.



The set of equations that correspond to this signal flow graph is **GATE-2008**

(a) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}\beta & -\gamma & 0\\ \gamma & \alpha & 0\\ -\alpha & -\beta & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1&0\\0&0\\0&1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

(b) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0 & \alpha & \gamma\\ 0 & -\alpha & -\gamma\\ 0 & \beta & -\beta\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0&0\\0&1\\1&0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

(c) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-\alpha & \beta & 0\\ -\beta & -\gamma & 0\\ \alpha & \gamma & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

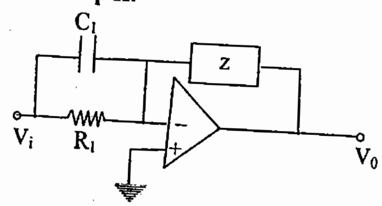
(d) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-\gamma & 0 & \beta\\ \gamma & 0 & \alpha\\ -\beta & 0 & -\alpha\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0&1\\0&0\\1&0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

55. The unit step response of an under damped second order system has steady value of −2. Which one of the following transfer functions has these properties?

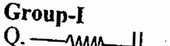
(a) $\frac{-2.24}{s^2+2.59s+1.12}$ (b) $\frac{-3.82}{s^2+1.91s+1.91}$

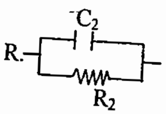
(c) $\frac{-2.24}{s^2-2.59s+1.12}$ (d) $\frac{-3.82}{s^2-1.91s+1.91}$

56. **Group I** gives two possible choices for the impedance Z in the diagram. The circuit elements in Z satisfy the condition $R_2C_2 > R_1C_1$. The transfer function $\frac{V_0}{V_i}$ represents a kind of controller. Match the impedances in **Group I** with the types of controllers in **Group II.**



Group-I | **Group-II**

Q.  **1.** PID controller



R. **2.** Lead compensator

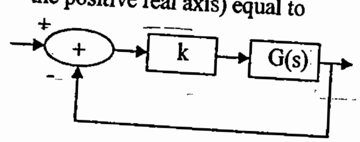
3. Lag compensator

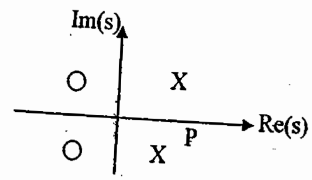
(a) Q-1, R-2 (b) Q-1, R-3
(c) Q-2, R-3 (d) Q-3, R-2

57. The feedback configuration and the pole-zero locations of

$$G(s)=\frac{s^2-2s+2}{s^2+2s+2}$$ are shown below.

The root locus for **negative** values of k, i.e, for −∞ < k < 0, has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to

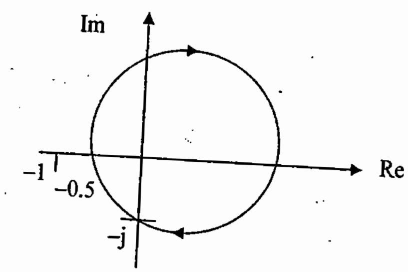


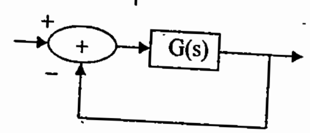


(a) $\pm\sqrt{2}$ and 0^0 (b) $\pm\sqrt{2}$ and 45^0

(c) $\pm\sqrt{3}$ and 0^0 (d) $\pm\sqrt{3}$ and 45^0

Common Data for Questions 58 & 59: The Nyquist plot of a stable transfer function G(s) is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown.





58. Which of the following statements is true?
(a) G(s) is an all-pass filter
(b) G(s) has a zero in the right-half plane
(c) G(s) is the impedance of a passive network
(d) G(s) is marginally stable

59. The gain and phase margins of G(s) for closed loop stability are
(a) 6 dB and 180^0 (b) 3 dB and 180^0
(c) 6 dB and 90^0 (d) 3 dB and 90^0

60. How many roots with positive real parts do the equation $s^3+s^2-s+1=0$ have ?
(a) Zero (b) One (c) Two (d) Three

END OF THE BOOK

SPACE FOR WORK